Part IIB - Economic Policy
Additional Notes

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Supervision 4: Government Debt Dynamics

Using definitions and accounting identities the following relationship is observed:

$$\Delta b = d + (r - g)b.$$

Typically economists will be interested in the dynamics of this relationship around the steady state value. These may be determined in two steps.

1. **Steady State**. We first work out the steady state level of b (denoted b^*) as:

   $$b^* = -\frac{d}{(r-g)},$$

   This infers $\Delta b = 0$, and is therefore the steady state level of debt-to-GDP for **any** $d \lesseqgtr 0$ **and any** $r \lesseqgtr g$ (provided $r \neq g$).

2. **Stability**. We next explicitly consider local stability. A stable (convergent) equilibrium will require that:

   $$\begin{aligned} \Delta b < 0 \quad &\text{if} \quad b > b^*, \\ \text{and} \quad \Delta b > 0 \quad &\text{if} \quad b < b^*. \end{aligned}$$

   To do this consider some small deviation, $\epsilon > 0$, around the steady state level, b^*. Specifically we have at the points $b = b^* \pm \epsilon$ that:

$$\begin{aligned}\Delta b &= d + (r-g)b, \\ &= d + (r-g)b^* + (r-g)\epsilon \\ &= (r-g)\epsilon,\end{aligned}$$

where we have used the form of the steady state. Note that to progress further we need to know the sign of $(r-g)$, with $r < g$ associated with stability. It is clear that a number of different cases will arise. These results are shown exhaustively in Table 1.

Table 1: Stability Comparison

Case	Deficit	$(r-g)$	$b^* = -\frac{d}{(r-g)}$	Consider deviations $\epsilon > 0$ $b = b^* + \epsilon$	$b = b^* - \epsilon$	Stable?
1	Deficit, $d > 0$	$r > g$	$b^* < 0$	$\Delta b = \epsilon(r-g) > 0$ **Unstable** (above)	$\Delta b = -\epsilon(r-g) < 0$ **Unstable** (below)	No, as both necessary conditions violated
2	Deficit $d > 0$	$r < g$	$b^* > 0$	$\Delta b = \epsilon(r-g) < 0$ **Stable** (above)	$\Delta b = -\epsilon(r-g) > 0$ **Stable** (below)	Yes, as both necessary conditions met
3	Surplus, $d < 0$	$r > g$	$b^* > 0$	$\Delta b = \epsilon(r-g) > 0$ **Unstable** (above)	$\Delta b = -\epsilon(r-g) < 0$ **Unstable** (below)	No, as both necessary conditions violated
4	Surplus, $d < 0$	$r < g$	$b^* < 0$	$\Delta b = \epsilon(r-g) < 0$ **Stable** (above)!	$\Delta b = -\epsilon(r-g) > 0$ **Stable** (below)!	Yes, as both necessary conditions met