The Impact of Large Scale Asset Purchases on Wealth Inequality*

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Abstract

In this paper I consider the impact of changes in the size and composition of the central bank’s balance sheet on household wealth inequality through the financial portfolio rebalancing channel of monetary policy transmission. A model with multiple assets (of differing liquidity) and heterogeneous agents, who experience idiosyncratic labour productivity shocks, is used to demonstrate that an expansion of the central bank’s balance sheet can materially alter the distribution of wealth, causing inequality to increase, while even extreme changes in the composition have little effect. When the model is calibrated to match the Federal Reserve’s Large Scale Asset Purchases, wealth inequality increases by 6.9%, as measured by the Gini coefficient.

JEL Codes: D31, E52, E58.

Keywords: Heterogeneous Agents, Large Scale Asset Purchases, Monetary policy.

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1 Introduction

In response to the global financial crisis, monetary policymakers used a variety of measures in an effort to stimulate aggregate demand. However, the channels through which more unconventional forms of monetary policy transmit to the real economy are widely debated, as are their efficacy and distributional consequences.

This paper investigates the impact of central bank Large Scale Asset Purchases (LSAPs) on an endogenous distribution of household wealth. The primary contribution to the literature is through the presentation of a theoretical model which can account for both financial income and portfolio transmission channels. This includes a set of heterogeneous households who face idiosyncratic risks due to labour productivity shocks and incomplete financial markets which prevent an equilibrium solution with full risk-sharing. The model is calibrated to match the distribution of US wealth prior to the global financial crisis, before changes to the central bank’s balance sheet are investigated.

My main finding is that the Federal Reserve’s LSAP programmes contributed to an increase in US wealth inequality through financial portfolio channels, raising the Gini coefficient by 6.9%. This occurred through quantitative easing programmes as central bank purchases of illiquid assets encouraged households to hold proportionally more of their portfolio as liquid wealth. As liquid wealth is less evenly distributed than illiquid wealth, various measures of inequality increase. The Federal Reserve’s maturity extension programme is found to slightly mitigate the impact of quantitative easing.

Various empirical effects of monetary policy shocks on wealth inequality has been estimated through a number of theoretical channels. In one channel Doepke and Schneider (2006b) show how higher inflation can reduce the real value of nominal debt contracts, reducing wealth inequality. In an alternative approach, Coibion et al. (2017) consider a range of inequality measures and find monetary policy shocks systematically increase inequality in labour earnings, total income, consumption and total expenditures.

Several recent contributions to the literature have highlighted the importance of analysing distributional effects when describing the transmission of monetary policy shocks. Kaplan et al. (2018) present a comprehensive comparison of the transmission mechanism in representative and heterogeneous agent models, while Sterk and Tenreyro (2018) use a balance sheet approach to model central bank open market operations to analyse monetary policy
transmission in an economy with agents of different ages. Broer et al. (2020) analyses a financial income channel of transmission, highlighting the role of profits in generating the standard increase in output following expansionary monetary policy shocks. The balance sheet approach used in this paper to specify monetary policy operations enriches these contributions by allowing variations in the size and composition of the central bank’s balance sheet to be investigated.

The remainder of the paper is structured as follows. Two stylised examples are presented in Section 2. The existing theoretical and empirical literature is then discussed in Section 3. Section 4 presents a theoretical model with household heterogeneity which explicitly accounts for the composition of the central bank’s balance sheet and Section 5 discusses the model calibration. Section 6 presents the main results of the paper. Section 7 concludes.

2 Stylised Example

This section presents two rudimentary methods to uncover the potential implications from asset price and quantity movements, consistent with those surrounding LSAPs. Both methods provide evidence to suggest LSAPs may have increased household wealth inequality.

2.1 Price Movements - Constant Allocations

Holding asset allocations constant, asset price movements have contributed toward the increase in wealth inequality since 2013. A counterfactual analysis can be conducted to determine the plausible change in wealth inequality as a result of asset price movements. This assumes fixed cross-sectional asset allocations such that asset price movements are the sole contributor to changes in wealth inequality. The procedure occurs in three steps. Initially a detailed picture of household wealth portfolios across the distribution of households is obtained from the 2007 US Survey of Consumer Finances (SCF).\textsuperscript{1} Next, price movements are assigned to each component of household wealth, following the methodology outlined in Kuhn and Rios-Rull (2016). This matches the individual components of household wealth to real house and stock market price movements, taken from Shiller (2015). Household wealth portfolios are re-evaluated for various population groups over time.

\textsuperscript{1} See appendix for further details on the US SCF.
The results of this exercise suggest wealth inequality has increased since the great recession, and can be seen in Figure 1, which plots the (indexed) real value of household wealth for the 2nd and 5th quintiles of the 2007 wealth distribution. During the great recession, a fall in both house and stock prices caused a fall in the value of wealth portfolios for both the 2nd and 5th quintiles. Initially the reduction in value of wealth is of a similar magnitude for both quintiles. However, between 2009 and 2012 the slower recovery in house prices, than in the stock market, combined with a greater portfolio allocation towards housing for households in the 2nd quintile of the net wealth distribution caused wealth valuations to diverge. A gap opens between the two valuations, indicating an increase in wealth inequality. Overall the reduction in the real value of wealth between 2007 and 2012 is of a similar magnitude to the increases seen in the early 2000s, for both quintiles

Figure 1: Pure Effect of Asset Prices on the Value of Net Wealth, by Quintile

Sources and Notes: US SCF (2007) and Shiller (2015), see Appendix A. January 2007 = 100. 5th quintile are the richest households, while the 2nd quintile are the second least wealthy.

Although this exercise is instructive in recovering the pure effect of asset price changes on inequality through changes to the value of wealth, it has severe limitations. The assumption of fixed asset portfolios is strong but difficult to overcome in empirical work. Moreover, as household rebalance their portfolios, the accuracy of these valuation effects reduces when moving further from the initial date. For these reasons, a theoretical model can help

\footnote{The 2nd quintile is used instead of the 1st to remove issues surrounding to negative wealth held by this group.}
to uncover general equilibrium effects, particularly those arising from portfolio rebalancing channel of transmission.

Nonetheless this exercise also provides a mechanism to enable a quick calculation which suggests in total the pure impact of real price changes over this period may be responsible for a 3% increase in the Gini coefficient and a 14% increase in the coefficient of variation. These back-of-the envelope calculations are shown in Table 1, which shows the impact of a change in property and stock market prices on measures of wealth inequality. In net, during the period between 2007 and 2017, real house prices fell by 15.6% while stock prices increased by 40%. Holding asset allocations fixed at the 2007 levels, both asset price changes increased the Gini coefficient and coefficient of variation. The impact of a fall in real house prices had a larger impact on the Gini coefficient, than the rise in stock prices. But the ordering is reversed when considering the coefficient of variation.

<table>
<thead>
<tr>
<th>% Price Change</th>
<th>Data (2007 SCF)</th>
<th>Property</th>
<th>Stocks</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gini</td>
<td>0.849</td>
<td>0.870</td>
<td>0.857</td>
<td>0.876</td>
</tr>
<tr>
<td>% change</td>
<td>2.5</td>
<td>0.9</td>
<td>3.1</td>
<td></td>
</tr>
<tr>
<td>Coef. of Var.</td>
<td>6.65</td>
<td>7.02</td>
<td>7.21</td>
<td>7.60</td>
</tr>
<tr>
<td>% change</td>
<td>5.7</td>
<td>8.5</td>
<td>14.3</td>
<td></td>
</tr>
</tbody>
</table>

Sources and Notes: US SCF (2007) and Shiller (2015), see Appendix A. Real price changes are taken as the 10 years between January 2007 and January 2017. These are then combined with a fixed asset allocation taken from the US SCF (2007) to calculate the impact on measure of wealth inequality.

### 2.2 Quantity Movements

Moving away from the data towards theory allow predictions to be made about the distribution of household wealth. In most modern economic models with heterogeneous households, the distribution of household wealth is an endogenous object. Researchers devote substantial time, effort and patience in calibrating this to closely match observed data though a variety of techniques.\(^3\) In so doing, the perspective of the distribution of household wealth often shifts towards becoming a fixed variable of interest - a target - rather than an outcome variable of interest. Research is then conducted by considering small, temporary, perturbations around this

\(^3\) This includes but is not limited to heterogeneous interest rates, heterogeneous discount factors and variations of a discretised stationary income process.
stationary point, perpetuating they myth that monetary policy shocks have a small impact on the distribution of household wealth. This conclusion rests on two assumptions, which are both violated when considering LSAPs, their size and the length of time over which central banks have maintained these balance sheet positions contribute to a reassessment of this position.

Typically, researchers investigate a one-time 100bps shock to the real interest rate. These are (unsurprisingly) insufficient to generate any meaningful change in the distribution of household wealth. In reality, best estimate suggest LSAPs represent around 400bps extra on the effective lower bound on nominal interest rates, thereby showing how the scale of these interventions is larger that the typical shock, and therefore will have larger impacts on the distribution of household wealth.

Persistence. Most analysis considers perturbations around a fixed (potentially stochastic) equilibrium. In reality the influence of LSAPs is still being felt, as central banks decide to run down their balances sheets naturally, rather than enacting outright sales.

These points are illustrated in a conventional Ayigari model, Figure 2 below, which is calibrated to match the observed US wealth distribution. From this initial position, a shock to the availability of assets is imposed. While the central bank provides additional funds to the government bond market, the government’s net demand for assets is therefore lower from the perspective of households and fewer household savings are required to meet this demand (Figure 2, Panel c). This lowers the real interest rate. The impact of this shock to effective asset supply on the distribution of household wealth is shown in Panels a and b. As fewer assets are available, the distribution of household assets shifts to the left. In equilibrium, more households face a binding liquidity constraint as wealth remains concentrated among the richest part of the population (who themselves hold fewer assets). This downwards shift in the number of assets held, combined with an increasing number of households at their borrowing constraint shift the Lorenz curve outwards. Inequality as measured by the Gini coefficient increases.
3 Related Literature

The related literature can be divided into several strands. The aggregate impact of LSAPs on asset prices, investors’ asset allocations, GDP and inflation is discussed first. A large empirical literature finds a strong statistical link between LSAPs and asset prices, and an associated portfolio rebalancing effect. It also finds a significant positive impact of LSAPs on GDP growth, and inflation. A second strand of the literature provides two essential tools to analyse the consequences of LSAPs in a theoretical model of wealth inequality. Both household heterogeneity and market frictions are required to model inequality and allow monetary policy to impact real variables. A final strand considers the relationship between inequality and monetary policy, where contributions are primarily focussed on conventional monetary policy. These three broad strands of the literature are now discussed in turn.
3.1 The Aggregate Impact of LSAPs

The empirical evidence suggests LSAPs have a significant impact on the price of a variety of assets. Two different empirical exercises dominate the literature. In the first, the authors construct short event windows around monetary policy announcements to capture the exogenous effect of policy announcements on asset prices. These typically show a strong response of the yields on government bonds, mortgage-backed securities, equity valuations and exchange rates. Although the precise effects differ depending on both the empirical methodology and specific LSAP programme, when scaled to a common size, LSAP programmes which purchase assets worth 10% of nominal GDP are usually found to reduce 10-year bond yields by 70-100bps in the US and UK, but by a far smaller amount in the euro area and Japan. These results are shown in Table 2.

Table 2: Impact of LSAPs on 10-year Government Bond Yields

<table>
<thead>
<tr>
<th>Study</th>
<th>Sample</th>
<th>Method</th>
<th>Yield Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>US</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Krishnamurthy and Vissing-Jørgensen (2011)</td>
<td>2008-2009</td>
<td>Event study</td>
<td>90</td>
</tr>
<tr>
<td>Krishnamurthy and Vissing-Jørgensen (2011)</td>
<td>2010-2011</td>
<td>Event study</td>
<td>90</td>
</tr>
<tr>
<td>Gagnon et al. (2011)</td>
<td>2008-2009</td>
<td>Event study</td>
<td>77 (60)</td>
</tr>
<tr>
<td>D’Amico et al. (2012)</td>
<td>2009-2010</td>
<td>Weekly time-series regression</td>
<td>170</td>
</tr>
<tr>
<td>D’Amico and King (2013)</td>
<td>2009-2010</td>
<td>Micro cross-section regression</td>
<td>144</td>
</tr>
<tr>
<td>Bauer and Rudebusch (2014)</td>
<td>2008-2009</td>
<td>Event study</td>
<td>75 (48)</td>
</tr>
<tr>
<td>Swanson (2016)</td>
<td>2009-2015</td>
<td>Factor model event study</td>
<td>79</td>
</tr>
<tr>
<td>Lloyd (2017)</td>
<td>2008-2012</td>
<td>Event study</td>
<td>117 (60)</td>
</tr>
<tr>
<td><strong>UK</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Joyce et al. (2011)</td>
<td>2009-2010</td>
<td>Event study</td>
<td>77</td>
</tr>
<tr>
<td>Joyce et al. (2012)</td>
<td>2009-2010</td>
<td>Event study, time series</td>
<td>75</td>
</tr>
<tr>
<td>Christensen and Rudebusch (2012)</td>
<td>2009-2011</td>
<td>Event study</td>
<td>34 (46)</td>
</tr>
<tr>
<td>McLaren et al. (2014)</td>
<td>2009,2012</td>
<td>Event study</td>
<td>65</td>
</tr>
<tr>
<td><strong>Euro Area</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Andrade et al. (2016)</td>
<td>2015-2016</td>
<td>Event study</td>
<td>41</td>
</tr>
<tr>
<td>De Santis (2020)</td>
<td>2004-2015</td>
<td>Error correction model</td>
<td>63</td>
</tr>
<tr>
<td><strong>Japan</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lam (2011)</td>
<td>2008-2010</td>
<td>Event study</td>
<td>9 (4)</td>
</tr>
</tbody>
</table>

Notes: Author’s calculations from reported values. Yield reduction is measured in basis points. Across studies, results are normalised to an LSAP programme with asset purchases worth 10% of nominal GDP. Parentheses indicate term prema effect, also measured in basis points.
The second empirical method uses dynamic term structure models to distinguish the effect of LSAP announcements on the risk premium and the expected path of future policy rates. These models find a smaller, yet significant, effect of LSAPs on the risk premium component of asset prices, for example reducing 10-year government bond yields by between 40-60bps in the US. Where applicable, these results are reported in parentheses in Table 2.

Further empirical work suggests investors adjust their asset allocations after central bank LSAPs. Carpenter et al. (2015) examine aggregate US Flow of Funds data over the recent financial crisis to identify the class of investor selling assets to the Federal Reserve, and how these investors rebalance their portfolios. They highlight the role of segmented financial markets, with heterogeneous asset sales to the Federal Reserve across investors. In the Treasury bond market, the central bank ultimately purchases bonds from households, who rebalance their portfolios towards corporate bonds, commercial paper and municipal debt.4

Analysing the effect in the UK, Joyce and Tong (2012) use a greater degree of disaggregation to assess the behaviour of individual life insurance companies and pension funds. In a counterfactual analysis, they show LSAP programmes induced similar portfolio rebalancing behaviour by both types of investor, who reduced their government bond asset allocations in favour of corporate bonds. Underlying this, a greater level of heterogeneity is displayed by life insurance companies, with the reallocation more pronounced for funds with greater equity weights.5 Although their results show strong portfolio rebalancing channels between government and corporate bonds, the impact of LSAPs on equity allocations is less clear, as institutional investors moved out of this asset class over the time horizon considered.

While central bank LSAPs appear to substantially influence financial market prices and allocations, their broader impact on economic activity and prices is less clear. The impact on aggregate measures of economic activity is critical in assessing the impact on wealth inequality, as although an initial shock to relative asset prices may be sufficient to alter wealth inequality, given heterogeneous asset positions, subsequent changes in aggregate variables may plausibly unwind these effects. Empirically, as aggregate variables are further removed from the initial shock, greater methodological assumptions are usually required to ascertain the broader impact of LSAPs. Nevertheless, the existing literature uses two main methods to uncover plausible effects. The first technique uses various restrictions on vector autoregres-

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4 The household category in the US Flow of Funds includes hedge funds.
5 Joyce and Tong (2012) consider these firms less risk averse.
sive (VAR) models to simulate counterfactual paths for GDP and inflation or present impulse response functions (IRFs). A second popular technique is to embed central bank asset purchases into a DSGE model and simulate shocks. Both methods generally show LSAPs to have a substantial positive impact on output and inflation, as presented in Table 3. Overall, the various Federal Reserve LSAPs are found to increase output and inflation, but the magnitude of this effect is unclear. The same is true in the UK, although the Bank of England’s LSAP programmes are generally found to have had a smaller impact. The more recent programmes in the euro area are found to have effects which are smaller still.

Table 3: Impact of LSAPs Aggregate Activity and Prices

<table>
<thead>
<tr>
<th>Study</th>
<th>Sample/Simulation</th>
<th>Method</th>
<th>Peak Impact (pp)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Output</td>
</tr>
<tr>
<td><strong>US</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baumeister and Benati (2013)</td>
<td>2009</td>
<td>VAR &amp; counterfactual</td>
<td>0.9</td>
</tr>
<tr>
<td>Gertler and Karadi (2013)</td>
<td>QE1</td>
<td>DSGE simulation</td>
<td>3.5</td>
</tr>
<tr>
<td>Chen et al. (2012)</td>
<td>QE2</td>
<td>DSGE simulation</td>
<td>0.1</td>
</tr>
<tr>
<td>Gertler and Karadi (2013)</td>
<td>QE2</td>
<td>DSGE simulation</td>
<td>1.0</td>
</tr>
<tr>
<td>Engen et al. (2015)</td>
<td>2009-2015</td>
<td>DSGE simulation (FRB/US)</td>
<td>0.5</td>
</tr>
<tr>
<td>Chung et al. (2012)</td>
<td>2010-2012</td>
<td>DSGE simulation (FRB/US)</td>
<td>3.0</td>
</tr>
<tr>
<td>Gambacorta et al. (2014)</td>
<td>2008-2011</td>
<td>VAR IRF</td>
<td>4.3</td>
</tr>
<tr>
<td><strong>UK</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baumeister and Benati (2013)</td>
<td>2009</td>
<td>VAR &amp; counterfactual</td>
<td>0.7</td>
</tr>
<tr>
<td>Joyce et al. (2011)</td>
<td>2009</td>
<td>VAR &amp; counterfactual</td>
<td>1.5</td>
</tr>
<tr>
<td>Kapetanios et al. (2012)</td>
<td>2009</td>
<td>VAR &amp; counterfactual</td>
<td>1.4</td>
</tr>
<tr>
<td>Gambacorta et al. (2014)</td>
<td>2008-2011</td>
<td>VAR IRF</td>
<td>3.9</td>
</tr>
<tr>
<td>Bridges and Thomas (2012)</td>
<td>2009-2011</td>
<td>Various</td>
<td>2.0</td>
</tr>
<tr>
<td>Pesaran and Smith (2016)</td>
<td>2009-2011</td>
<td>ARDL &amp; counterfactual</td>
<td>1.0</td>
</tr>
<tr>
<td>Weale and Wieladek (2016)</td>
<td>2009-2014</td>
<td>VAR IRF</td>
<td>3.1</td>
</tr>
<tr>
<td><strong>EA</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lenza et al. (2010)</td>
<td>2008-2010</td>
<td>VAR &amp; counterfactual</td>
<td>2.5*</td>
</tr>
<tr>
<td>Gambacorta et al. (2014)</td>
<td>2008-2011</td>
<td>VAR IRF</td>
<td>1.4</td>
</tr>
<tr>
<td>Gambetti and Musso (2020)</td>
<td>2009-2016</td>
<td>VAR &amp; counterfactual</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Notes: Author’s calculations from reported values. Results are not normalised, with initial shocks varying. In Kapetanios et al. (2012), Pesaran and Smith (2016) and Bridges and Thomas (2012) model averages are reported. The peak impact in Weale and Wieladek (2016) multiplies the peak impulse response by the largest policy announcement shock. Estimates in Gambacorta et al. (2014) use central country-specific impulse estimates and observed balance sheet changes. *The output estimate in Lenza et al. (2010) uses industrial production.

Comprehensive reviews of the impact of recent central bank LSAP programmes, as well as other unconventional monetary policy measures, can be found in both Borio and Zabai (2018) and Haldane et al. (2016).
3.2 Tools to Analyse the Heterogeneous Impact of LSAPs

Two analytical devices are required to study the impact of monetary policy on wealth inequality. Firstly, any model requires heterogeneous agents and incomplete financial markets. Secondly, market frictions are also necessary to enable monetary policy to have real effects.

The standard model of heterogeneous agents with incomplete financial markets has undergone a series of transformations since first being proposed by Bewley (1980, 1983, 1986). Huggett (1993) analyses a partial equilibrium with trade in a zero net supply asset, while Aiyagari (1994) extends this analysis to a general equilibrium setting, for a production economy. The interaction between heterogeneous households and incomplete financial markets in these models generates an intrinsic precautionary saving motive, as households save to self-insure against possible future adverse idiosyncratic states. Further work by Krusell and Smith (1998) introduces a concept of bounded rationality to permit solutions for models with additional aggregate uncertainty resulting from aggregate business cycle productivity shocks.

Comprehensively matching moments of the household wealth distribution is challenging, even in a model with incomplete financial markets and both idiosyncratic and aggregate productivity risks. Typically the strength of the precautionary saving motive declines as wealth increases, relative to labour income. This results in the poorest households holding more wealth than the data suggests, while the richest households hold less.\(^6\) Krueger et al. (2016) note how these differences bias idiosyncratic household impulse responses towards those from representative agent models and propose increasing the persistence of earnings risk, to substantially increase wealth dispersion.\(^7\) In most modifications to the standard model of heterogeneous agents with incomplete markets, financial markets remain limited to a single risk-free one-period bond, which is insufficient to study the effects of Large Scale Asset Purchases, which requires multiple assets and the possibility of portfolio rebalancing.

A second tool required to analyse the impact of monetary policy on wealth inequality are market frictions. In his influential Jackson Hole paper Woodford (2012) highlighted the need for market frictions to ensure central bank LSAPs have a real effect on the economy, as a specific case of the general Wallace (1981) irrelevance result. The general result shows

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\(^6\) In Carroll (1997) households stop saving once they reach a buffer-stock of capital, beyond which richer households have a negative saving rate.

\(^7\) A similar solution is also suggested in Carroll et al. (2015) who add buffer-stock saving to the standard Krusell and Smith (1998) model, and Ravn and Sterk (2017), where search and matching frictions in the labour market generate this persistence through additional unemployment risk.
that central bank open market operations have no effect on the real economy when accompanied by fiscal transfers ensuring an invariant income distribution. A reallocation of assets between economic agents has no economic impact if assets are only valued for their payoffs and budget constraints are the sole restriction on investors’ portfolio positions. Market frictions are therefore required for monetary policy to have real effects and the literature suggests a variety of forms these could take, each capturing a particular feature of the monetary transmission mechanism. Eggertsson and Woodford (2003) highlight the role of expectations, while Gertler and Kiyotaki (2010) explicitly model disruptions to financial intermediation to highlight the central bank’s role in providing liquidity support to the financial sector. However, when considering the impact of LSAPs, the most appropriate market friction to break the Wallace (1981) irrelevance result should focus on the portfolio rebalancing channel of monetary transmission, as this most clearly distinguishes LSAPs from other forms of monetary policy.

Two model varieties focus on the portfolio rebalancing channel of monetary policy transmission. In one approach, investors have preferred-habitat demand preferences for assets with differing term structures. Vayanos and Vila (2021) show how the preferred-habitat view of the term structure can generate asset demand which differs substantially from no-arbitrage restrictions, with the government bond term structure instead determined through an interaction between investors with preferences for bonds of specific maturities and risk averse arbitrageurs. In an alternative approach, Andrés et al. (2004) introduce a model with portfolio adjustment costs and exogenous financial market participation restrictions to model the separate impact of monetary policy on short- and long-term assets. This allows them to separate the impact of open market operations on long-term assets through the expected path of short-term rates and an additional influence of monetary policy on relative prices through term premia. Chen et al. (2012) calibrate preferred-habitat, adjustment costs and exogenous financial market participation restrictions to a medium-scale DSGE model and simulate the impact of Federal Reserve LSAPs. They find little aggregate impact when matching the low degree of market segmentation observed in the data.

3.3 The Relationship Between Inequality and Monetary Policy

The literature identifies several potential channels of transmission for monetary policy shocks to influence the distribution of household wealth. In an important contribution, Auclert (2019)
analytically decomposes the response of heterogeneous households into three broad classes (inflation, earnings and interest rate exposure). He then shows how variation along each dimension can generate different marginal propensities to consume from a monetary policy shock. Refining his latter two categories, the literature can be split into five channels of transmission with the net impact of monetary policy shocks on wealth inequality depending both on the balance between these channels of transmission and also on the extent to which the response of income inequality translates over time into a wealth effect. In response to an expansionary monetary policy shock these are as follows:

1. **Inflation and savings redistribution.** An (unanticipated) increase in inflation following a monetary policy shock erodes the real value of nominal debt contracts, thereby transferring wealth from savers to borrowers, reducing wealth inequality. The empirical relevance of this transmission channel is explored extensively by Doepke and Schneider (2006b), who find a ‘moderate’ inflationary surprise (a 5pp increase, sustained for 10 years) leads to a sizeable redistribution of wealth. The largest gains (from 2.6 to 5.9 percent of GDP) accrue to middle-class households under age 45, who hold substantial fixed-rate mortgage positions. In contrast, middle-class retirees (over age 65) lose most as inflation erodes the real value of savings. This inflationary surprise episode also causes a material redistribution from the household sector to the government, the primary net debtor to the household sector. In a theoretical contribution, Akyol (2004) examines the welfare implications of different steady state rates of inflation, showing that in an incomplete markets setting the Friedman rule (setting the nominal interest rate to zero) is not optimal. Instead, when money is useful for consumption smoothing, the monetary authority may use a small positive level of inflation to achieve a higher level of social insurance, as a tax on households who receive large endowment shocks with seigniorage revenue evenly distributed. The benefit of this, relative to self-insurance, diminishes as inflation increases.

2. **Labour supply channel.** Theoretically, households should re-optimize their labour supply decisions in an attempt to mitigate the impact of a change in relative asset prices, resulting from an expansionary monetary policy shock. However, empirical estimates of this facet of household behaviour (the intertemporal substitution effect)

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8 In related work, Doepke and Schneider (2006a) also show such a zero-sum wealth redistribution may generate persistent aggregate effects.
vary substantially. Although most estimates suggest expansionary monetary policy reduces inequality through labour channels, the empirical approach usually abstracts from decomposing the equilibrium effect into labour supply and demand channels. For example, by combining Romer and Romer (2004) exogenous monetary policy shocks with household-level consumer expenditure data, Coibion et al. (2017) find monetary policy shocks alter US earnings inequality by a similar order of magnitude as their impact on GDP and inflation. A 100bps expansionary conventional monetary policy shock is found to decrease income inequality by 1.5% as measured by the Gini coefficient. In a related paper focussing on the UK, Mumtaz and Theophilopoulou (2017) find similar results.

3. **Labour demand channel.** Expansionary monetary policy shocks are usually associated with higher aggregate demand and additional job creation. In an extensive summary of the differences in monetary policy transmission between representative and heterogeneous agent models, Kaplan et al. (2018) argue the endogenous response of indirect channels, including labour demand and fiscal policy, are responsible for over 75% of the first quarter consumption response, compared to under 5% in a typical representative agent model. They show real interest rate movements and intertemporal substitution effects primarily stem from these indirect effects. Lütticke (2021) draws similar conclusions in a model calibrated to replicate business cycle statistics. This indirect aggregate channel of transmission is also important as central bankers have argued the positive impact of LSAPs on job creation absolves any medium-term influence of these policies on wealth inequality as low skilled workers should benefit more from lower unemployment.⁹

The primary empirical sources for the impact of monetary policy on labour market channels are reported together in Table 4.

---

⁹ For example Haldane (2014).
Table 4: Impact of Monetary Policy Shocks on Income Inequality

<table>
<thead>
<tr>
<th>Country</th>
<th>Study</th>
<th>Inequality Measure</th>
<th>Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>Coibion et al. (2017)</td>
<td>Gini coefficient</td>
<td>1.5%</td>
</tr>
<tr>
<td>UK</td>
<td>Mumtaz and Theophilopoulou (2017)</td>
<td>Gini coefficient</td>
<td>3%-10%</td>
</tr>
<tr>
<td>Euro Area</td>
<td>Guerello (2017)</td>
<td>Gini coefficient</td>
<td>1%</td>
</tr>
<tr>
<td>Panel</td>
<td>Furceri et al. (2018)</td>
<td>Gini coefficient</td>
<td>2%</td>
</tr>
</tbody>
</table>

Notes: Author’s calculations from reported values of the cumulative response, scaled to a +100bps movement in the monetary policy instrument against income inequality. Studies use a similar VAR methodology with the exception of Furceri et al. (2018) who use local projection.

4. **Financial wealth channel.** Given that households hold different amounts of financial assets, the capital income boost from an expansionary monetary policy shock results in heterogeneous relative income shocks. Wealthier households typically hold a larger proportion of wealth as financial assets and therefore experience a larger income rise through this channel. Broer et al. (2020) focus on this aspect of the monetary transmission mechanism using a version of the standard New Keynesian model with capitalists and workers to highlight the role of profits in generating the standard increase in output following expansionary monetary policy shocks. Coibion et al. (2017) also present factor income heterogeneity as an important transmission channel. Empirically, they find aggregate labour income responds little to monetary policy shocks while financial and business income fluctuate extensively. A second component of within this channel arises as an increase in the price of financial assets following an expansionary monetary policy shock causes wealth inequality to increase through valuation effects which fall upon the individuals who are most closely connected to financial markets.

5. **Relative asset price channel.** After accounting for the total size of households’ financial assets, the extent of any financial income boost also depends on asset allocations, with heterogeneous portfolios also determining the response of wealth inequality to monetary policy shocks. As monetary policy alters relative asset prices households observe disproportionate shocks. These portfolio balance sheet effects are examined by Domanski et al. (2016) in a partial equilibrium setting (abstracting from portfolio rebalancing) using household finance surveys for a range of countries. They find an increase in equity valuations since the global financial crisis has benefited the wealthiest members of society, increasing wealth inequality, with a more even distribution from
house price gains only partially offsetting this. Doepke et al. (2015) also conclude that the house price channel of transmission is not large enough to mitigate the impact on wealth inequality of inflationary shocks.

The empirical results from the asset allocation channel of transmission are not universally accepted. In a similar analysis O’Farrell et al. (2017) find monetary policy shocks to be an insignificant determinant of changes in net wealth inequality.

Overall the effect of an expansionary monetary policy shock on wealth inequality largely depends on the balance between lower inequality through labour channels and a greater level of inequality which arises as a result of financial channels.

Existing theoretical and empirical models have limited commentary on unconventional monetary policy, although some include brief discussions of forward guidance. Instead, the models discussed above generally rely on a monetary policy rule specifying the central bank’s reaction function, without specifically modelling open market operations and balance sheet changes. This point is also noted by Sterk and Tenreyro (2018), a paper highly related to this one. Sterk and Tenreyro (2018) specifically focus on modelling central bank balance sheet policies, and suggest two distinct channels of monetary policy transmission towards heterogeneous households: through inflation altering the value of real wealth (channel 1, above), and through an associated change in transfers to households which occur as a result of remittance income from the central bank to the treasury. An important contribution they make is to model different sectors of the economy, concluding that consumers substitute towards durable goods in response to expansionary monetary policy shocks. Sterk and Tenreyro (2018) introduce household heterogeneity through an overlapping generations model, which naturally focuses on monetary policy transmission across demographic groups within a life-cycle structure.

This paper contributes to the literature in two regards. Firstly, it presents a theoretical model which can account for financial income and portfolio transmission channels. It then calibrates the model to match recent US LSAPs to show these forms of unconventional monetary policy have increased wealth inequality through these financial channels.
4 Model

The model features five agent types: households, firms, banks, the government and a central bank. Heterogeneous households will largely follow a classical Bewley-Huggett-Aiyagari style model, extended to include durable goods, taken to largely represent each household’s stock of housing wealth. The central bank’s balance sheet will be modelled in detail enabling different LSAP programmes to be defined theoretically. Firms are modelled simply and, to cleanly distinguish the impact of policy, neither the government or central bank are engaged in optimising behaviour.

Notation is as follows: Upper (lower) case variables denote nominal (real) variables, but on two occasions upper case variables denote stocks (H and B). Subscripts firstly denote the agent; \(i\) indicates household-specific variables, while \(g\) and \(s\) denote the supply of a variable from the fiscal and monetary authority respectively; \(b\) indicates a bank. Second subscripts denote the time period. Superscripts are used to denote the asset to which a net rate of return, proportion, price or demand refers.

4.1 Households

The household side of the model closely follows the durables goods extension to Guerrieri and Lorenzoni (2017). A continuum of heterogeneous households (with unit mass) maximise their expected lifetime utility:

\[
\max_{(c_{i,j}, d_{i,j+1}, h_{i,j+1})_{j=t}^\infty} \mathbb{E}_t \sum_{j=t}^{\infty} \beta^{j-t} u(c_{i,j}, h_{i,j+1}, 1 - \ell_{i,j}),
\]

where \(\beta \in (0, 1)\) is the household discount factor and the time-separable, concave and increasing period-\(t\) felicity function, \(u\), for household \(i\) depends on their individual level of real consumption, \(c_{i,t}\), the flow utility gathered from housing which is taken to be proportional to the stock, \(h_{i,t}\), and labour supply, \(\ell_{i,t}\), which is normalised to be within the unit interval. As in Castañeda et al. (2003), households are assumed to care equally about their own utility and that of their descendants. Utility maximisation occurs subject to a series of period budget constraints. Stated in real terms, for household \(i\), the period-\(t\) budget constraint is:

\[
c_{i,t} + d_{i,t+1} + p_t h_{i,t+1} + \phi(h_{i,t+1}, h_{i,t}) = (1 + r_d^t) d_{i,t} + p_t^h (1 - \delta) h_{i,t} + \tau_{i,t} + w t \zeta_{i,t} \ell_{i,t}.
\]

17
The left-hand side of the budget constraint represents the household’s expenditure. Each period households use available resources to purchase consumption, $c_{i,t}$, deposit real money balances with retail banks, $d_{i,t+1}$, and purchase illiquid housing stock, $h_{i,t+1}$ at its relative price, $p_{i}^{h}$. Finally, agents must pay a real cost of adjusting their stock of illiquid asset holdings, $\phi(\cdot)$, which is assumed to be convex with $\phi(h_{i,t+1}, (1 - \delta)h_{i,t}) = 0$ and $\phi(\cdot) \geq 0$, such that adjustment costs are paid only when housing stock is adjusted by an amount other than its rate of depreciation, given by the parameter $\delta$. The right-hand side of the budget constraint represents household’s income. Purchases are financed using the gross real return from previously invested bank deposits, $(1 + r_{t}^{d})d_{i,t}$, housing assets which may be sold to fund purchases, $p_{i}^{h}(1 - \delta)h_{i,t}$, idiosyncratic transfers, $\tau_{i,t}$ (discussed further below), and finally labour income, $w_{t}\zeta_{i,t}\ell_{i,t}$, which itself depends on an idiosyncratic productivity level, $\zeta_{i,t}$ and the real wage $w_{t}$. A subset of retired households receive an idiosyncratic productivity level $\zeta_{i,t} = 0$, such that their optimal decision will be to supply no labour.

Households face two further constraints on their choices: a collateral constraint for bank deposits and a liquidity constraint for housing stock:

$$
(1 + r_{t+1}^{d})d_{i,t+1} \geq -\Phi_{d} p_{i}^{h}(1 - \delta)h_{i,t+1}, \quad (3a)
$$
$$
h_{i,t+1} \geq 0, \quad (3b)
$$

When binding, these constraints limit household portfolio choices. The first constraint states that household banking debt must be collateralised by those housing assets available to make repayments in future periods. $\Phi_{d}$ therefore represents the fraction of housing assets available as collateral to household borrowing, with the restriction $\Phi_{d} < 1$ used to guarantee households may repay in full. In addition, in this representation $\Phi_{d}$ controls how frequently the contralateral constraint binds for bank deposits, which may represent an adjustable rate bank loan, renegotiated each period. Large bank loans are therefore similar to adjustable rate mortgages (ARMs). In this setting, idiosyncratic labour income risk prevents uncertain future labour being used as collateral for borrowing. The second constraint normalises the minimum level of permissible housing stock. Taken together, incomplete financial markets therefore introduce a precautionary saving motive for heterogeneous households who seek to self-insure against adverse labour productivity shocks.
At this stage it is useful to represent the real value of households’ net wealth, \( x_{i,t} \), as:

\[
x_{i,t} \equiv (1 + r_d^d) d_{i,t} + p_i^h (1 - \delta) h_{i,t},
\]

such that, suppressing indexation for each individual, the household problem may be rewritten in recursive form, with the Hamilton-Jacobi-Bellman equation:

\[
v(x, h, \zeta) = \max_{c, d', h'} \left\{ u(c, h, 1 - \ell) + \beta \mathbb{E}[v(x', h', \zeta')] \right\},
\]

subject to the same constraints:

\[
c + d' + p_i^h h' + \phi(h', h) = x + \tau + w \zeta \ell.
\]

\[
x' = (1 + r_d^d) d' + p_i^h (1 - \delta) h',
\]

\[
x' \geq (1 - \Phi_d^d) p_i^h (1 - \delta) h',
\]

\[
h' \geq 0,
\]

For computational purposes, it is important to note the households’ state variables. For household \( i \) these consist of total asset holdings, \( x_{i,t} \) and the composition of this which are in illiquid housing assets, \( h_{i,t} \), idiosyncratic productivity level, \( \zeta_{i,t} \), and the set of aggregate price and quantity variables. The cross-sectional joint distribution of asset holdings and productivity is also a state variable, and can be represented by the cumulative distribution function, \( F_t(\zeta_{i,t}, d_{i,t}, b_{i,t}) \). Shocks to idiosyncratic labour productivity are assumed to be persistent, and are discussed further below.

### 4.2 Firms

Production technology is assumed to be symmetric across the consumption and housing services markets. Firms are perfectly competitive and have access to a linear production function in labour. In terms of the consumption goods price, the firms problem is therefore:

\[
\max_{n^c_i} z_i^c n_i^c - w_t n_i^c,
\]

\[
\max_{n^h_i} p_i^h z_i^h n_i^h - w_t n_i^h,
\]
such that the first order conditions determine both the real wage and relative price of housing as:

\[ w_t = z_t^c, \]
\[ p_t^h = \frac{z_t^c}{z_t^h}, \]

where both may then be calibrated directly.

### 4.3 Government

Fiscal policy is not the focus of this paper, and a number of simplifying assumptions are made. The government focuses on debt issuance, as fiscal policy is formed without government spending, or distortionary taxation, with net transfers to households made on a lump-sum basis.\(^\text{10}\) The period-\(t\) nominal government budget constraint can be written as:

\[
A_{g,t+1} + V_{t}^{B} (B_{g,t+1} - B_{g,t}) + S_t = (1 + R_t^A) A_{g,t} + B_{g,t} + P_t T_t. \tag{10}
\]

The fiscal authority has access to two financing options: a one period nominal coupon bond, \(A_{g,t}\), and an infinitely-lived (perpetual annuity) nominal consol, \(B_{g,t}\). The current price of the one-period bond is taken as the numeraire, while the end-of-period (post-coupon) value of a consol is \(V_{t}^{B}\). The budget constraint states that the value of new government debt issuance and the transfer income received from the central bank, \(S_t\) (discussed further below), must equal the value of outstanding debt obligations and transfers made to households, \(T_t \equiv \int \tau_{i,t} dF_t\), which are explained further below. One-period coupon bonds yield a net return of \(R_t^A\), while each consol pays one unit of currency in every time period. Newly issued consols, which start paying coupons in the next period, have the same market value as the post-coupon value of those issued previously, such that the value of new consol issuance is given as the second term in equation 10. The budget constraint can be re-written in real terms:

\[
\frac{A_{g,t+1}}{P_t} + \frac{V_{t}^{B} B_{g,t+1}}{P_t} + \frac{S_t}{P_t} = \frac{(1 + R_t^A) A_{g,t}}{P_t} + \frac{(1 + V_{t}^{B}) B_{g,t} V_{t-1}^{B}}{P_t V_{t-1}^{B}} + T_t. \tag{11}
\]

\(^\text{10}\) The analysis here closely follows Harrison (2012) and Sterk and Tenreyro (2018), who explore this exposition in greater detail.
\[ a_{g,t+1} + b_{g,t+1} + s_t = (1 + r^a_t) a_{g,t} + (1 + r^b_t) b_{g,t} + T_t, \]  

(12)

where the second line uses the definition of real value and rate of return for the one-period coupon bond, while the real value of the consol and its real price are tracked over time and analogously defined. Together these definitions are:

\[ a_{g,t+1} \equiv \frac{A_{g,t+1}}{P_t}, \]  

(13a)

\[ 1 + r^a_t \equiv \frac{(1 + R^A_t) P_{t-1}}{P_t}, \]  

(13b)

\[ b_{g,t+1} \equiv \frac{V^B_t B_{g,t+1}}{P_t}, \]  

(13c)

\[ 1 + r^b_t \equiv \frac{(1 + V^B_t) P_{t-1}}{P_t V^B_{t-1}}. \]  

(13d)

In this way, the characterisation of long-term government debt as consols ensures that the rate of return depends only on the change in their relative price over a single time period. In real terms, the constraint (12) states that government payments on previously issued debt, and transfers to households are funded through new government debt issuance or remittance income from the central bank.

Implicit in this representation of central government constrained behaviour is a coordination with the central bank, such that the central government does not simply offset changes in the relative holdings of the central bank’s assets by altering its debt issuance, a point particularly highlighted in Kaplan et al. (2018). This is not the case in practise, as debt management offices do not alter debt issuance as a result of monetary policy action. To ensure against this case, when solving the model, the supply of each asset, \( A_{g,t+1} \) and \( B_{g,t+1} \), will be calibrated at constant values to match the data.

### 4.4 Central Bank

Although no frictions are assumed to exist between the central bank and treasury, a conceptual distinction is made between the fiscal and monetary authorities. The central bank’s balance sheet is now modelled explicitly, with a graphical representation shown in Figure 3. Each period the central bank issues nominal reserve balances, \( M_{s,t+1} \). As the economy is cashless, reserve balances also represent the measure of base money. Against this liability, the central bank holds one-period liquid government nominal coupon bonds. It holds a propor-
tion, $\lambda_{t+1}^a \in [0, 1)$, of the total supply. The central bank also holds a proportion, $\lambda_{t+1}^b \in [0, 1)$, of the total supply of government issued infinitely-lived nominal consols.\footnote{It can be assumed that a practical upper bound exists for both $\lambda_{t+1}^a$ and $\lambda_{t+1}^b$ below 1. The Bank of Japan are closest to reaching a practical upper limit, with an asset purchase programme twice the size of new government issuance, though still hold only 40% of the total outstanding stock of Japanese Government Bonds. It is also assumed $\lambda_{t+1}^a \lambda_{t+1}^b > 0$ to ensure $M_{s,t+1} > 0$.} In real terms, at the start of period-$t$ the value of the central bank balance sheet can therefore be written as:

$$M_{s,t} = \lambda_t^a A_{g,t} + \lambda_t^b V_{t} B_{g,t}. \tag{14}$$

Holding prices and government debt issuance constant, an expansion of the monetary base is associated with an increase in either $\lambda_{t+1}^a$ or $\lambda_{t+1}^b$.

The central bank’s earns income on assets held from the previous period, and pays a nominal rate of return, $R_t^M$, on central bank issued reserve balances. Each period, all net central bank income is assumed to be given directly to the fiscal authority as seigniorage payments, $S_t$, which can then be defined as:

$$S_t = (1 + R_t^A) \lambda_t^a A_{g,t} + (1 + V_t^B) \lambda_t^b B_{g,t} - (1 + \hat{M}_{s,t+1} + R_t^M) M_{s,t}, \tag{15}$$

where $\hat{M}_{s,t+1}$ is the percentage change in nominal central bank issued reserve balances:

$$\hat{M}_{s,t+1} \equiv \frac{M_{s,t+1} - M_{s,t}}{M_{s,t}}. \tag{16}$$

Central bank seigniorage payments revenue may be rewritten in real terms:

$$s_t = (r_t^a - r_t^m) \lambda_t^a a_{g,t} + (r_t^b - r_t^m) \lambda_t^b b_{g,t} + \mu_t m_{s,t}, \tag{17}$$

where definitions consistent with (13a - 13d) are used for real central bank issued reserve
balances, \( m_{s,t} \), and their real rate of return, \( r_{t}^m \).

\[
m_{s,t+1} \equiv \frac{M_{s,t+1}}{P_t}, \quad (18a)
\]

\[
1 + r_{t}^m \equiv \frac{(1 + R_t^M)P_{t-1}}{P_t}, \quad (18b)
\]

\[
\mu_t \equiv \frac{\hat{M}_{s,t+1}P_{t-1}}{P_t}, \quad (18c)
\]

\[
\mu_t m_{s,t} = \frac{M_{s,t+1} - M_{s,t}}{P_{s,t}}. \quad (18d)
\]

The final term, \( \mu_t m_{s,t-1} \), is the traditional seigniorage expression, which would appear in a model without payments on either assets or liabilities. Two other sources of seigniorage revenue arise as a result of differences between the rate of return on central bank base money and either one-period nominal coupon bonds or government consols. For a given balance sheet size, seigniorage payments can always be made positive if the nominal rate of return on central bank issued reserve balances is sufficiently small.

Assuming a constant supply of assets, \( A_{g,t} \) and \( B_{g,t} \), and constant asset prices, three different stylised central bank policies are now considered:

1. In an initial steady state, the central bank holds a constant fraction of both one-period liquid real government bonds and illiquid assets. In addition, the central bank’s supply of base money is unchanged. The real size of the balance sheet is therefore given as:

\[
m_{s,t} = \lambda_t^a a_{g,t} + \lambda_t^b b_{g,t}. \quad (19)
\]

A stylised example of this is shown in the first panel of Figure 3, where the real size of the balance sheet in this initial steady state is normalised to 100. The contribution of illiquid assets to the central bank’s balance sheet is small, with \( \lambda_t^a a_{g,t} \gg \lambda_t^b b_{g,t} \). The assumptions of these policies for monetary policymakers’ decision variables are shown in Table 5.
Notes: This figure shows real assets and liabilities of the central bank’s balance sheet in three stylised scenarios. In the initial steady state the central bank holds predominately liquid government assets, \(\lambda^a_t a_{g,t} \gg \lambda^b_t b_{g,t}\).

Under a Quantitative Easing programme (QE) the central bank purchases illiquid assets, \(\lambda^b_{t+1} b_{g,t+1}\), expanding the base money supply, \(m_{s,t+1}\). Under a Maturity Extension Programme (MEP) the central bank alters the composition of the asset-side of the balance sheet, purchasing illiquid assets and selling liquid assets. Liabilities are unchanged after a MEP.

2. Under a Quantitative Easing programme (QE), the central bank purchases a large amount of illiquid assets, funded by creation of additional base money. Holdings of liquid assets are unchanged. An example of this can be seen by moving from the first to the second panel of Figure 3, where the real size of the balance sheet doubles, with the increase on the asset-side of the balance sheet entirely due to the increase in illiquid assets.

3. Under a Maturity Extension Programme (MEP), the central bank restructures the asset-side of its balance sheet, without increasing the overall size. The central bank’s liquid asset holdings fall to offset an increase in illiquid assets (a sterilised open market operation). As a result of this programme \(\lambda^a_{t+1} a_{g,t+1} \ll \lambda^b_{t+1} b_{g,t+1}\). This can be seen by moving from the second to the final panel in Figure 3. The implications of this policy for monetary policymakers’ decision variables are shown in the final column of Table 5.
4.5 Retail Banking Sector

The banking sector is modelled simply. A number of retail banks operate under perfect competition, taking prices as given. Retail banks hold liquid government debt and central bank issued reserve balances as assets, while they hold household deposits and bank capital, $k_{b,t+1}$ as liabilities. The end of period-$t$ balance sheet of retail banks can be written as:

$$a_{b,t+1} + m_{b,t+1} = d_{b,t+1} + k_{b,t+1}. \quad (20)$$

Retail banks are also subject to a regulatory capital requirement:

$$\frac{k_{b,t+1}}{k_{b,t+1} + d_{b,t+1}} \geq \omega, \quad (21)$$

where $\omega \in [0, 1)$ represents the retail bank’s capital-to-asset ratio. Retail banks are assumed to be risk neutral and seek to maximise their franchise value equivalent to the present discounted value of expected profits, subject to their balance sheet and regulatory capital requirements:

$$\max_{\{a_{b,t+1}, m_{b,t+1}, d_{b,t+1}, k_{b,t+1}\}} \mathbb{E}_t \sum_{j=t}^{\infty} \beta^{j-t} \pi_{b,j}, \quad (22)$$

where period-$t$ profits are given as:

$$\pi_{b,t} = (1 + r^a_t)a_{b,t} + (1 + r^m_t)m_{b,t} - (1 + r^d_t)d_{b,t} - k_{b,t+1}. \quad (23)$$

After observing that the capital constraint will bind in equilibrium, retail bank capital can be eliminated from the problem. The three remaining first order conditions of the maximisation can be written as:

$$1 + r^a_{t+1} = 1 + r^m_{t+1} = (1 + r^d_{t+1})(1 - \omega), \quad (24)$$
where the first equality results from the assumption that retail banks can costlessly switch funding source between central bank reserve balances and liquid government debt. These conditions show how the central bank simultaneously controls the expected real rate of return on reserve balances, liquid government debt and household deposits, where the latter is deflated to account for the capital requirement rule.

4.6 Transfer Payments

To close the model a transfer payment scheme is required which maps ownership of retail bank capital and infinitely-lived (perpetual annuity) nominal government consols to households. At the aggregate level government transfer payments are given as:

\[ T_t = (1 + r^b_t)(1 - \lambda^b_t)b_{g,t} - (1 - \lambda^b_{t+1})b_{g,t+1}, \]

which are then distributed to households as in the following two-part form:

\[ \tau_{i,t} = \begin{cases} \frac{T_t}{\int_{\zeta_{i,t}=0} \theta dF_t} + \pi_{b,t} & \text{if } \zeta_{i,t} = 0, \\ \pi_{b,t} & \text{otherwise}, \end{cases} \]

such that the government infinitely-lived nominal consols purely to finance state pension payments for households in their retirement state with \( \zeta_{i,t} = 0 \) (appropriately scaled for this population sub-group), while household ownership of bank capital is spread evenly throughout the population such that each household receives an equal share of retail bank profits, \( \pi_{b,t} \). In steady state households receive no income from this latter source due to perfect competition in the retail banking sector and the assumption of a constant balance sheet size, \( k_{b,t+1} = k_{b,t} \).

4.7 Equilibrium Definition

A recursive stationary equilibrium is defined in each time period as a set of policy functions, aggregate prices \( r^a_t, r^m_t, r^d_t, w_t, p^h_t \); and aggregate quantities, \( n^c_t, n^h_t, a_{g,t}, b_{g,t}, s_t, T_t \); such that:

1. Households’ policy functions maximise utility, given any aggregate pricing vector.
2. Aggregating across individual policy functions generates feasible allocations, which clear all markets at the equilibrium aggregate pricing vector such that:

\[ \int c_i \, dF_t = z^c_t n^c_t, \]  
\[ \int h_i \, dF_t = z^h_t n^h_t, \]  
\[ \int \zeta_i \ell_i \, dF_t = n_i + n^h_i, \]  
\[ \int d_i \, dF_t = (1 - \omega)(a_{g,t} + \lambda^b_i b_{g,t}), \]  
\[ m_{s,t} = m_{b,t}, \]  

respectively referring to equilibrium conditions for the goods, housing services, labour and retail deposit markets as well as the market for central bank reserve balances.

3. The distribution of agents, \( F_t(\zeta_{i,t}, d_{i,t} - 1, b_{i,t} - 1) \), is invariant.

Taken together, this characterisation of the economy results in aggregate equilibrium balance sheet positions as given in Table 6.

<table>
<thead>
<tr>
<th>Agent</th>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Government</td>
<td>( \text{NPV}(s_t - T_t) )</td>
<td>( a_{g,t} + b_{g,t} )</td>
</tr>
<tr>
<td>Central Bank</td>
<td>( \lambda^a_t a_{g,t} + \lambda^b_t b_{g,t} )</td>
<td>( m_{s,t} )</td>
</tr>
<tr>
<td>Financial Sector</td>
<td>( (1 - \lambda^a_t) a_{g,t} + m_{s,t} )</td>
<td>( d_{b,t} )</td>
</tr>
<tr>
<td>Households</td>
<td>( d_{b,t} + (1 - \lambda^b_t) b_{g,t} + p^h_i h_i + k_{b,t} )</td>
<td>( 0 )</td>
</tr>
</tbody>
</table>

Notes: \( \text{NPV}(\cdot) \) calculates the net present value.

## 5 Calibration & Computation

Before the model can be solved, a functional form of the period-\( t \) felicity function is chosen:

\[ u(c_{i,t}, h_{i,t}, 1 - \ell_{i,t}) = \frac{(d^{i}_{i,t} h^{1-\sigma}_{i,t})^{1-\sigma} - 1}{1 - \sigma}, \]  

where the coefficient of relative risk aversion, \( \sigma > 0 \), is chosen to ensure strict convexity of marginal utility with respect to consumption such that prudence is positive, ensuring house-
holds display precautionary saving behaviour. In order to focus on financial transmission and balance sheet effects, the utility trade-off between leisure and labour supply is normalised, with households each supplying the same number of hours worked, $\ell_{i,t} = 0.7$, in equilibrium.\(^{12}\) The parameter $\theta$ captures the expenditure share of consumption, up to the presence of the collateral constraint.\(^{13}\)

A functional form of the real adjustment costs is also required. This is taken from the investment literature as:

$$
\phi(h_{i,t+1} - h_{i,t}) = \frac{\phi_1}{2} \left( \frac{h_{i,t+1} - (1 - \delta)h_{i,t}}{h_{i,t}} \right)^2 h_{i,t},
$$

where $\phi_1 > 0$, to ensure that households do not pay adjustment costs when their housing stock only falls by depreciation.

### 5.1 Calibration

The model calibration procedure occurs in three parts. The first sets parameters equal to those generally found in the literature, where possible. A second set of parameters are set equal to their natural empirical counterparts. A final set of parameters are calibrated targeting moments of the initial wealth distribution, closely following the procedure outlined in Castañeda et al. (2003). Table 7 summarises the baseline calibration, with parameters of particular interest or sensitivity to the results described in greater detail below. The model is parametrised at a quarterly frequency with GDP normalised to 1 with GDP-ratios equal to aggregate variables.

The majority of the parameters calibrated to those found in the literature are standard, with the housing services expenditure share and depreciation rate being chosen to match Guerrieri and Lorenzoni (2017).

Among the most important empirically estimated variables are the aggregate level of ex-

\(^{12}\) Lütticke (2021) demonstrates how this may be done implicitly through the use of Greenwood et al. (1988) preferences.

\(^{13}\) The elasticity of substitution between consumption and housing services is assumed to be 1, following the literature. A number of papers attempt to estimate the elasticity of housing substitution with little consensus. Macroeconomic studies generally find a parameter value consistent with housing and non-housing consumption being substitutes, while studies at the household level find these distinct components of consumption to be complements. For example Davis and Martin (2009) provide estimates using aggregate data and ? using household-level data. For this reason most articles use assume an elasticity of 1 and investigate properties around this. Davis and Ortalo-Magné (2011) show the expenditure share of housing services to be stable over both time and metropolitan statistical areas in the United States.
Table 7: Baseline Calibration

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Literature</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time-discount factor</td>
<td>$\beta$</td>
<td>0.98</td>
<td>Standard value</td>
</tr>
<tr>
<td>Coefficient of relative risk aversion</td>
<td>$\sigma$</td>
<td>2.00</td>
<td>Standard value</td>
</tr>
<tr>
<td>Housing expenditure share*</td>
<td>$1 - \theta$</td>
<td>0.30</td>
<td>Guerrieri and Lorenzoni (2017)</td>
</tr>
<tr>
<td>Housing depreciation rate</td>
<td>$\delta$</td>
<td>1.3%</td>
<td>Guerrieri and Lorenzoni (2017)</td>
</tr>
<tr>
<td><strong>Empirical</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Government real supply of short-term liquid assets</td>
<td>$a_{g,t}$</td>
<td>0.08</td>
<td>SIFMA issuance data</td>
</tr>
<tr>
<td>Initial real value of government illiquid assets</td>
<td>$b_{g,t}$</td>
<td>0.25</td>
<td>SIFMA issuance data</td>
</tr>
<tr>
<td>Aggregate productivity (consumption)</td>
<td>$z^c_t$</td>
<td>1.43</td>
<td>$y_t = 1$</td>
</tr>
<tr>
<td>Aggregate productivity (housing services)</td>
<td>$z^h_t$</td>
<td>1.44</td>
<td>$p^h_t = 1$</td>
</tr>
<tr>
<td>Bank capital ratio</td>
<td>$\omega$</td>
<td>0.60</td>
<td></td>
</tr>
<tr>
<td><strong>Targeted Moment</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adjustment cost coefficient</td>
<td>$\phi_1$</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>Collateral availability</td>
<td>$\Phi^d$</td>
<td>0.8</td>
<td>Guerrieri and Lorenzoni (2017)</td>
</tr>
</tbody>
</table>

Notes: *For households for whom the collateral constraint does not bind.

ogenous liquid, $a_{s,t}$, and illiquid, $b_{s,t}$, real asset supply. Data on the size of the outstanding US Treasury market are taken from the Securities Industry Financial Markets Association (SIFMA). Short-term, liquid government assets are taken to be US Treasury Bills. For the 10-year period prior to the financial crisis, between 1996 and 2006, these averaged 8% of nominal GDP. Illiquid assets are taken to be Treasury Notes and Bonds (assets with a remaining maturity above 2 years). An analogous exercise shows these illiquid assets average 25% of nominal GDP in the 10 years prior to the financial crisis.

Policy variables for the central bank holdings of liquid, $\lambda^c_t$, and illiquid, $\lambda^b_t$, government securities are calibrated by combining the SIFMA data with Federal Reserve balance sheet data from the Federal Reserve Bank of New York. This is done in several stages, with the SIFMA data first interpolated to a monthly frequency to generate monthly estimates of $\lambda^c_t$ and $\lambda^b_t$. Three month averages are then taken around specific dates of interest with adjustments made to ensure consistency in the scale of the Federal Reserves’ balance sheet. The first adjustment assumes all assets held in the TAF are liquid, the second adjustment assumes all other non-Treasury asset purchases by the Federal Reserve over this period are illiquid assets.\textsuperscript{14} The results of this exercise are shown in Table 8. The impact of individual LSAP programmes may then be evaluated, with the overall impact taken as the difference between the initial (pre-QE1) steady state and the final (post-QE3) steady state.

\textsuperscript{14} During this step $\lambda^b_t$ is increased to account for Federal Reserve MBS purchases.
Table 8: Policy Calibration

<table>
<thead>
<tr>
<th>Policy</th>
<th>Date</th>
<th>$\lambda_a^t$</th>
<th>$\lambda_b^t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-QE1 (Initial)</td>
<td>Nov 2008</td>
<td>0.31</td>
<td>0.10</td>
</tr>
<tr>
<td>Post-QE2</td>
<td>Jun 2011</td>
<td>0.07</td>
<td>0.32</td>
</tr>
<tr>
<td>Pré-MEP</td>
<td>Sep 2011</td>
<td>0.10</td>
<td>0.34</td>
</tr>
<tr>
<td>Post-MEP</td>
<td>Dec 2012</td>
<td>0.00</td>
<td>0.33</td>
</tr>
<tr>
<td>Pré-QE3</td>
<td>Aug 2012</td>
<td>0.01</td>
<td>0.34</td>
</tr>
<tr>
<td>Post-QE3 (Final)</td>
<td>Oct 2014</td>
<td>0.00</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Sources and Notes: SIMFA and FRB New York. $\lambda_a^t$ and $\lambda_b^t$ respectively refer to the proportion of total liquid and illiquid government asset supply held by the central bank. Initial value of $\lambda_a^t$ is adjusted to allow for the TAF, discussed above. $\lambda_b^t$ are adjusted to allow for non-Treasury illiquid asset purchases.

Given the average level of idiosyncratic labour productivity is normalised at $\bar{\zeta}_{i,t} = 1$, further details below, and labour supply is normalised at $\ell_t = 0.7$, the level of aggregate productivity in the consumption sector is set to ensure aggregate output is normalised to $y_t = w_t \int \zeta_{i,t} \ell_{i,t} dF_t = 1$, which is achieved by setting $w_t = z_{c,t}^c = \frac{1}{\bar{\zeta}_{i,t} \ell_t} = \frac{10}{7} \approx 1.43$.

The value of housing stock is calibrated to match an aggregate share of net housing wealth of 30% aggregate household wealth, which is supported by data from both the SCF and US Flow of Funds prior to the financial crisis. The level of housing productivity, $z_{h,t}^h$, and aggregate goods production, $z_{c,t}^c$, are then used to re-normalise aggregate output.

The parameter pertaining to the retail banking sector, $\omega$, is used to calibrate an initial share of liquid assets of 11.5%, to match SCF data.

5.1.1 Income Process

The final empirically estimated parameters concern the idiosyncratic productivity process and have significant implications for the model solution, as individuals facing small, infrequent shocks have little incentive for precautionary saving behaviour. The calibration procedure for the income process therefore warrants special attention, as this will be vital to achieving a realistic starting level for the distribution of household wealth, from which to investigate the impact of LSAPs. Key to delivering the required result is allowing household to have both the ability and desire to generate a distribution of household assets which greatly resembles the data. To this end, the calibration procedure captures two of the most important features of the income distribution, both displayed in Figure 4, which shows the income of household heads taken from the 2005 PSID. Two features are striking. The first is the life-cycle struc-
ture of earnings, which increase rapidly towards middle age before falling back dramatically for households who retire at ages around 65. This hump shape generates a first motive for households borrowing when young, before saving for retirement during working life. A second feature of the data highlights heterogeneity through different productivity levels, proxied by college attendance. Household heads who attended (will attend during the sample) college have higher income throughout the cross-section.

![Figure 4: Life-cycle of labour income.](image)

Sources and Notes: PSID. Household heads only.

These two features are perhaps the most important features in generating a realistic income process, and are therefore taken directly from the data, which is split into two income groups and three life stages.

### 5.1.2 Income Process

This paper follows the literature in assuming log-labour productivity is assumed to follow a first order autoregressive process:

$$\ln \zeta_{i,t} = \rho \ln \zeta_{i,t-1} + \varepsilon_{i,t}, \quad (30)$$

where a discretised productivity states, $\zeta_{i,t}$, and their Markov transition matrix are used to target the distribution of household wealth following Castañeda et al. (2003), with the outcome parameters given in Table 9. Alongside this productivity process or employed households, a
life-cycle structure is imposed on the model following the literature in imposing an earnings persistence across generations.

<table>
<thead>
<tr>
<th>ζ_{i,t}</th>
<th>Π_{i,t,t+1}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.81</td>
<td>0.91 0.05 0.01 0</td>
</tr>
<tr>
<td>1.69</td>
<td>0.05 0.88 0.05 0</td>
</tr>
<tr>
<td>1.69</td>
<td>0.01 0.05 0.91 0</td>
</tr>
<tr>
<td>6.56</td>
<td>0 0.01 0.05 0.91</td>
</tr>
</tbody>
</table>

The parameter Φ^d is set to enable household to borrow 20% of the average value of liquid assets, matching the SCF data. Φ^b is set to 0, ensuring in this economy, all borrowing occurs in liquid assets. This simplification is consistent with the empirical results showing the proportion of the population with negative liquid net wealth is close to three times larger than that with negative illiquid net wealth.

The remainder of the calibration procedure uses the remaining parameters to match moments of the wealth distribution, accomplishing this by establishing a tight link between the model baseline model calibration and the data, as shown in Figure 5. One object of interest is the Gini coefficient taken from the 2004 SCF, as 0.816. Other non-targeted moments including the proportion of households with negative net worth, taken here to mean the proportion of hand-to-mouth households (H2M) and the proportion of the population holding negative liquid assets (d_{i,t} ≤ 0) are close to the observed data. The results of this calibration are shown in Table 10.

<table>
<thead>
<tr>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gini</td>
<td>0.82</td>
</tr>
</tbody>
</table>
5.2 Computation

Determining a steady-state equilibrium for this model is computationally intensive, primarily because the numerical procedure requires a simultaneous solution for the household portfolio choice problem alongside aggregate equilibrium prices. The main elements of the algorithm are described in both Judd (1998) and Heer and Maussner (2009). The procedure involves three key stages for any given level and composition of the central bank’s balance sheet.

In the first stage, aggregate prices are guessed, the household problem is solved, and policy functions obtained through value function iteration. A policy function iteration accelerator block is added to aid convergence speed.

The second stage requires convergence of the ergodic household distribution, \( F_t \). When asset grids for deposits and illiquid securities are combined with the discretised idiosyncratic productivity variables, an exact probability measure can be constructed across households. When this probability density measure is combined with the household policy functions, Matlab’s sparse matrix routine can be used to iteratively approximate the household distribution, \( F_t \) directly controlling numerical errors. Solving for the ergodic distribution is therefore direct, without simulation error.

In the final stage, a Bisection method is used to converge on the initial guess for the
aggregate pricing vector, ensuring market clearing conditions are met. For steady states with exogenous labour supply a double Bisection is generated over \( r^a \) and \( r^b \).

### 6 Results

The impact of LSAP programmes conducted by the Federal Reserve are now investigated. Steady state solutions for aggregate variables using the baseline model are given in Table 11. Many of these aggregate variables are obtained by construction as a result of the calibration process. This includes the normalisation of aggregate output, \( y \), and effective hours worked, \( n \). Although when different balance sheets are investigated central bank reserve balances, retail bank deposit holdings, \( d_t \), and housing stock, \( h_t \), will change from their initial steady state, the remaining aggregate variables will not change in the baseline specification.

<table>
<thead>
<tr>
<th>Table 11: Steady State Aggregate Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
</tr>
<tr>
<td>1.00</td>
</tr>
</tbody>
</table>

#### 6.1 Overall Effect of US LSAPs

From this initial position, the impact of the Federal Reserve’s LSAP policies on the aggregate household balance sheet is shown in the left hand panel of Table 12. Relative to the initial position, LSAPs decrease total household financial savings by 2.8%, which is comprised of two different effects. Households hold fewer illiquid assets, as their holdings of housing assets decreases substantially when the central bank expands its balance sheet by purchasing additional illiquid assets. This is offset by a substantial increase in liquid retail bank deposits of 83% as base money expands the size of retail banks’ balance sheets. The total effect remains substantially smaller due to the relative size of liquid and illiquid assets.

The primary result of this paper is that Federal Reserve LSAP programmes have increased wealth inequality through financial transmission channels. This can be seen in the right hand panel of Table 12. The Gini coefficient and coefficient of variation both increase as a result of

---

15 Asset grids each contain 50 nodes. Value function iteration converges when the norm of the change in value function is below \( 1 \times 10^{-9} \). Similarly, the distribution, \( F_t \), is taken to have converged when the norm of change in the probability statistic falls below \( 1 \times 10^{-9} \). Bisection tolerance is set to 1% of supply along each dimension.
these asset purchase programmes. The total impact is substantial, with the Gini coefficient increasing by 6.9%, and the coefficient of variation by 8.1%. Other measures of household wealth inequality, including the skewness, are also found to increase.

Table 12: Impact of US LSAPs, Household Sector

<table>
<thead>
<tr>
<th>Financial Balance Sheet</th>
<th>Measures of Inequality</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid Deposits</td>
<td>Illiquid Housing</td>
</tr>
<tr>
<td>Pre-QE1 (Initial)</td>
<td>0.68</td>
</tr>
<tr>
<td>Post-QE2</td>
<td>1.03</td>
</tr>
<tr>
<td>Pre-MEP</td>
<td>1.06</td>
</tr>
<tr>
<td>Post-MEP</td>
<td>1.05</td>
</tr>
<tr>
<td>Pre-QE3</td>
<td>1.06</td>
</tr>
<tr>
<td>Post-QE3 (Final)</td>
<td>1.23</td>
</tr>
<tr>
<td>Total change</td>
<td>81.0</td>
</tr>
</tbody>
</table>

Notes: Total change is taken from initial to final and reported in per cent.

Even large changes to the central bank’s balance sheet are found to have a moderate impact on steady state asset prices, shown in Table 13. In each variant of the central bank’s balance sheet, the real interest rate on liquid asset holdings is calculated. However, the difference between steady states is small. Comparing the Initial and Final steady states, as the central bank increases demand for illiquid assets, and reduces demand for liquid financial assets the real interest rate on liquid assets falls by 100bps. As household shift to an equilibrium of holding relatively more wealth in liquid assets, the relative price of deposits falls, with an associated increase in their real interest rate. The size of this change is somewhat surprising, given the 81% increase in household ownership of liquid financial assets, but in part reflects the loss of self-insurance through fewer illiquid asset holdings.
Table 13: Impact of US LSAPs, Real Interest Rates

<table>
<thead>
<tr>
<th></th>
<th>( r^d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-QE1 (Initial)</td>
<td>2.49</td>
</tr>
<tr>
<td>Post-QE2</td>
<td>3.29</td>
</tr>
<tr>
<td>Pre-MEP</td>
<td>3.48</td>
</tr>
<tr>
<td>Post-MEP</td>
<td>3.48</td>
</tr>
<tr>
<td>Pre-QE3</td>
<td>3.48</td>
</tr>
<tr>
<td>Post-QE3 (Final)</td>
<td>3.53</td>
</tr>
<tr>
<td><strong>Total change</strong></td>
<td><strong>1.04</strong></td>
</tr>
</tbody>
</table>

Notes: All interest rates are annualised percentages. \( r^d \) denotes the real interest rate on household deposits and household illiquid assets. Total change is taken from initial to final and reported in pp.

The changes in liquid interest rates implied by the model are positive, as opposed to the partial equilibrium estimates from the empirical literature. Estimates for the change in the term premia component presented in Table 2 suggest a fall in the real interest rate of illiquid assets by around 100bps = 20pp \( \times 0.1 \times 50\) bps as a result of the 20pp increase in the ratio of the central bank’s balance sheet to nominal GDP observed in the data. However, these partial equilibrium changes occur along the transition path between steady states suggesting the dynamics of transition may be useful to understand in greater detail. In contrast, the 104bps increase in the real interest rate reflects an increase in the premium associated with financial assets due to their risk-sharing capabilities, now that the central bank holds a greater proportion of the illiquid financial assets.

### 6.2 Effect by US LSAP programme

When the impact of Federal Reserve’s LSAP programmes on household wealth inequality is investigated, higher wealth inequality is found to arise solely through quantitative easing programmes, being only partially offset by the Federal Reserve’s maturity extension programme. This can be seen most evidently in Table 12. All three measures of wealth inequality (the Gini coefficient, coefficient of variation and skewness) fall as a result of the Federal Reserve’s MEP. In contrast, the QE programmes reinforce one another, increasing measures of household wealth inequality.

The mechanism under the MEP works in the opposite direction to that of the QE programmes. Even though, under MEP, the central bank holds a greater share of illiquid gov-
ernment consols, a valuation effect causes the value of consols held by households increase. Although the return on household deposits is little changed under MEP, the increase in the return on government consols is consistent with the mechanism in MEP working in the opposite direction to that under QE.

Although the Federal Reserve’s maturity extension programme reduces wealth inequality, this effect is marginal compared to the total impact of quantitative easing. Under MEP the Gini coefficient of household wealth falls slightly from 0.81 to 0.80, representing a 0.64% fall. In contrast, the impact of quantitative easing policies are found to be an order of magnitude higher, with the period between QE1 and QE2 found to increase the Gini coefficient from 0.77 to 0.80, a 4.1% increase, and QE3 found to increase the Gini coefficient from 0.81 to 0.82, a 2.2% increase.

7 Conclusion

This paper presents a model of the economy capable of distinguishing the impact of central bank LSAPs on an endogenous distribution of household wealth. Using a balance sheet approach to model the central bank, changes in monetary policy are found to alter wealth inequality significantly, as the distribution of household wealth transitions to a new steady state.

Once the model is calibrated to fit US data, it highlights the contribution of Federal Reserve LSAP programmes to the increase in wealth inequality over this period. This arises as the policy of quantitative easing is found to increase household wealth inequality which is only partially mitigated by the Federal Reserve’s maturity extension programme. In net Federal Reserve LSAPs increase wealth inequality by 6.9% as measured by the Gini coefficient. A broader consideration of standard statistical measures of the household wealth distribution also suggests these balance sheet policies increased household wealth inequality. This endogenous increase in measures of wealth inequality is consistent with the empirical evidence since the great recession.

Further research could seek to decompose the effects of central bank LSAPs on wealth inequality into various channel of transmission, in particular by differentiating the labour market response from that of the financial income channel. Theoretically, allowing the labour market channel of transmission to operate is expected to reduce the change in wealth in-
equality resulting from expansionary monetary policy, but the scale to which these channels may alleviate the financial effects is currently unclear. In the current version of this paper, the initial calibration of household wealth requires further attention.
References


Appendices

A  Data Description

A.1  Wealth and Assets Survey

The UK Wealth and Assets Survey (WAS) collects data on assets, liabilities and the distribution of net wealth from a sample of over 30,000 households. The survey is conducted in biannual waves, the first of which started in 2006. The WAS is published by the UK Office for National Statistics (ONS).

A.1.1  Additional Tables

Although the main text focuses on the US context, a similar ranking of inequality measures holds for in the UK data, with physical and housing asset the most evenly distributed throughout the population, while pension and financial assets are less evenly distributed. Table 14 shows composite measures of wealth inequality, by subcomponent of net wealth for the US and UK. These data show the US to have a higher absolute level of wealth inequality, which holds in all components. The substantially higher inequality of US households’ pension wealth, compared to the UK is indicative of different institutional arrangements. Figures 6 and 7, which show the same data plotted as Lorenz curves.

<table>
<thead>
<tr>
<th></th>
<th>Physical</th>
<th>Housing</th>
<th>Pension</th>
<th>Financial</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>US</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gini</td>
<td>0.58</td>
<td>0.88</td>
<td>0.90</td>
<td>0.97</td>
<td>0.86</td>
</tr>
<tr>
<td>Coef. of Var.</td>
<td>4.54</td>
<td>4.97</td>
<td>4.52</td>
<td>9.03</td>
<td>7.07</td>
</tr>
<tr>
<td><strong>UK</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gini</td>
<td>0.45</td>
<td>0.66</td>
<td>0.73</td>
<td>0.91</td>
<td>0.63</td>
</tr>
<tr>
<td>Coef. of Var.</td>
<td>1.11</td>
<td>1.88</td>
<td>2.00</td>
<td>18.84</td>
<td>3.17</td>
</tr>
</tbody>
</table>

A.2 Survey of Consumer Finances

Household wealth data are primarily taken from the US Survey of Consumer Finances (SCF), published by the Federal Reserve. This survey is ideally suited to the study of wealth inequality using household balance sheets, as it includes a stratified sample of the wealthiest households whose data would be difficult to match using standard sampling techniques. The SCF is conducted triennially and currently comprises around 6,000 US households, starting in the most comparable form in 1989. Households are usually interviewed between May and December of each survey year. In the current form households are only included for one wave, with the re-interview of households for the 2009 panel an exceptional case.

The calculation of net worth uses the methodology of Kuhn and Ríos-Rull (2016), matching their results with few differences. Where small differences exist these are likely accounted for by data truncation in the public record, not affecting Kuhn and Ríos-Rull (2016). Following this methodology, wealth is calculated at a household level, with a definition that includes the value of financial and real assets of all kinds, net of debts. This includes property; businesses (including farms); checking accounts, certificates of deposit, and other banking accounts; IRA/Keogh accounts, money market accounts, mutual funds, bonds and stocks, cash and call money at the stock brokerage, and all annuities, trusts, and managed investment accounts; vehicles; the cash value of term life insurance policies and other policies; money

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16 For example their Table 2 (Quantiles of the 2013 earnings, income, and wealth distributions) and Table 4 (Concentration and skewness decomposition).
owed to others; pension plans accumulated in accounts; and other assets. Debts include the value of property debts (including mortgages, home equity, and HELOCs); credit card debts; instalment loans; loans taken against pensions; loans taken against life insurance; margin loans and other miscellaneous debts.

Whenever percentiles are used a 100 household average around the percentile of interest is taken to reduce cross-sectional noise, bracketing the percentile of interest by approximately 1pp. Where small differences between the 2007 panel and main survey exist, these are likely accounted for by the exclusion of households who were not re-interviewed in 2009. Where real values are reported, nominal wealth is deflated using the PCE deflator.

A.2.1 Additional Tables

The evolution of US household wealth (net worth) is shown in Table 15. Over the past quarter of a century, total nominal US household wealth has increased more than fivefold. However the evolution of aggregate household wealth masks a substantial degree of heterogeneity. For instance, households in the 90th percentile of today’s wealth distribution are over 12 times wealthier than the median household, with a net worth of $1.2mn, compared to $93,400 at the 50th percentile. Heterogeneity among households is therefore important in describing the evolution of wealth. When examined by wealth percentile, the high proportion of wealth accruing to the top of the wealth distribution can be seen. Indeed, in nominal terms, those at the bottom of the wealth distribution are less wealthy today, than in 1989, with negative net worth.

Table 15: US Household Net Wealth ($ thousand)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>-2.1</td>
<td>-3.0</td>
<td>-2.1</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>2.0</td>
<td>3.0</td>
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<td>38.7</td>
<td>47.9</td>
<td>51.0</td>
<td>60.7</td>
<td>37.4</td>
<td>35.7</td>
<td>46.4</td>
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<td>46.1</td>
<td>48.0</td>
<td>56.0</td>
<td>71.3</td>
<td>84.2</td>
<td>90.8</td>
<td>115.7</td>
<td>73.1</td>
<td>77.2</td>
<td>93.4</td>
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<td>77.6</td>
<td>84.2</td>
<td>108.7</td>
<td>136.7</td>
<td>152.7</td>
<td>195.3</td>
<td>132.4</td>
<td>140.8</td>
<td>164.7</td>
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<tr>
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<td>112.6</td>
<td>116.2</td>
<td>123.9</td>
<td>167.8</td>
<td>224.1</td>
<td>250.8</td>
<td>294.5</td>
<td>229.9</td>
<td>233.2</td>
<td>271.1</td>
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<td>80</td>
<td>184.9</td>
<td>182.8</td>
<td>189.9</td>
<td>268.2</td>
<td>370.6</td>
<td>428.3</td>
<td>486.2</td>
<td>400.0</td>
<td>410.7</td>
<td>478.2</td>
</tr>
<tr>
<td>90</td>
<td>351.2</td>
<td>341.7</td>
<td>356.1</td>
<td>494.8</td>
<td>731.2</td>
<td>827.4</td>
<td>869.2</td>
<td>918.0</td>
<td>919.4</td>
<td>1150.9</td>
</tr>
<tr>
<td>Total ($ trillion)</td>
<td>16.3</td>
<td>20.0</td>
<td>21.0</td>
<td>29.7</td>
<td>42.9</td>
<td>50.2</td>
<td>59.3</td>
<td>64.3</td>
<td>86.7</td>
<td></td>
</tr>
</tbody>
</table>

Sources: US SCF.
These data also highlight how unusual changes in the wealth distribution have been since the financial crisis. Until 2007, nominal household wealth had been increasing for every decile in the distribution. Yet, since then, wealth has fallen for every decile other than the wealthiest. Households at the lower end of the wealth distribution saw proportionally larger declines in net wealth. The least wealthy households substantially increased the nominal value of debt. For example, in 2016 households at the 10th percentile had a negative net worth of $2,100. In contrast, the wealthiest households experienced rapid growth in net worth over the same period. Overall, total nominal household wealth had not recovered to its previous peak by 2013, but far exceeded this by 2016.

The composition of US household net worth across SCF samples is shown in Table 16. The various sub-components are then constructed to form a measure of household liquid and illiquid assets. Since 2007 the share of household liquid assets in total wealth has increased, from 11% to 14%, while that of illiquid assets has fallen, from 89% of net worth to 86%. At the same time the percentage of the population with negative liquid asset holdings has fallen, while the percentage of the population with negative illiquid assets has risen.

Table 16: Composition of Aggregate Household Net Wealth

<table>
<thead>
<tr>
<th>Year</th>
<th>Liquid Wealth</th>
<th>Illiquid Wealth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumer Debt</td>
<td>-1.8</td>
<td>-1.1</td>
</tr>
<tr>
<td>Deposits</td>
<td>14.1</td>
<td>12.5</td>
</tr>
<tr>
<td>Total Liquid</td>
<td>12.3</td>
<td>11.4</td>
</tr>
<tr>
<td>Negative (%)</td>
<td>34.3</td>
<td>33.0</td>
</tr>
<tr>
<td>Housing</td>
<td>38.6</td>
<td>45.4</td>
</tr>
<tr>
<td>Vehicles</td>
<td>3.6</td>
<td>3.8</td>
</tr>
<tr>
<td>Pension</td>
<td>3.2</td>
<td>3.1</td>
</tr>
<tr>
<td>Financial</td>
<td>12.0</td>
<td>11.0</td>
</tr>
<tr>
<td>Business</td>
<td>23.1</td>
<td>20.7</td>
</tr>
<tr>
<td>Other</td>
<td>7.1</td>
<td>4.6</td>
</tr>
<tr>
<td>Total Illiquid</td>
<td>87.7</td>
<td>88.6</td>
</tr>
<tr>
<td>Negative (%)</td>
<td>10.7</td>
<td>11.1</td>
</tr>
</tbody>
</table>

Sources and Notes: US SCF. Reported in percentage of total net worth. *Other is a residual category, which includes student debt.

These measures of liquid and illiquid wealth are used to calculate the contribution of each
to the underlying change in the wealth distribution over this period. They can also be used to
analyse the change in proportion of net wealth held in illiquid and liquid assets by households
at different points in the wealth distribution. This is shown in Table 17 which reports a stable
share of wealth held in illiquid assets over time, and throughout much of the cross-sectional
distribution. The changes since 2007 are therefore particularly notable. For households above
the 20th percentile of the wealth distribution, the proportion of net worth held in illiquid
assets have been at, or around, historic lows since 2007. For those households at below the
20th percentile of the wealth distribution interpretation of these data is more difficult, due to
the prevalence of liquid debt.

Table 17: Proportion of Wealth Held in Illiquid Assets, by Net Wealth Percentile

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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<th></th>
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</thead>
<tbody>
<tr>
<td>10</td>
<td>1.27</td>
<td>-0.45</td>
<td>0.44</td>
<td>-0.01</td>
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<tr>
<td>20</td>
<td>1.08</td>
<td>0.97</td>
<td>0.95</td>
<td>1.20</td>
<td>0.96</td>
<td>0.93</td>
<td>1.23</td>
<td>1.12</td>
<td>1.17</td>
<td>0.73</td>
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<tr>
<td>30</td>
<td>1.02</td>
<td>0.91</td>
<td>0.89</td>
<td>0.94</td>
<td>0.86</td>
<td>0.95</td>
<td>0.91</td>
<td>1.23</td>
<td>0.84</td>
<td>0.95</td>
</tr>
<tr>
<td>40</td>
<td>0.92</td>
<td>0.94</td>
<td>0.86</td>
<td>0.89</td>
<td>0.87</td>
<td>0.90</td>
<td>0.91</td>
<td>0.82</td>
<td>0.90</td>
<td>0.94</td>
</tr>
<tr>
<td>50</td>
<td>0.88</td>
<td>0.92</td>
<td>0.94</td>
<td>0.91</td>
<td>0.84</td>
<td>0.94</td>
<td>0.94</td>
<td>0.91</td>
<td>0.80</td>
<td>0.90</td>
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<tr>
<td>60</td>
<td>0.90</td>
<td>0.89</td>
<td>0.89</td>
<td>0.88</td>
<td>0.88</td>
<td>0.91</td>
<td>0.87</td>
<td>0.87</td>
<td>0.86</td>
<td>0.89</td>
</tr>
<tr>
<td>70</td>
<td>0.85</td>
<td>0.85</td>
<td>0.89</td>
<td>0.85</td>
<td>0.85</td>
<td>0.87</td>
<td>0.86</td>
<td>0.84</td>
<td>0.81</td>
<td>0.78</td>
</tr>
<tr>
<td>80</td>
<td>0.83</td>
<td>0.82</td>
<td>0.86</td>
<td>0.81</td>
<td>0.79</td>
<td>0.81</td>
<td>0.79</td>
<td>0.79</td>
<td>0.78</td>
<td>0.84</td>
</tr>
<tr>
<td>90</td>
<td>0.83</td>
<td>0.81</td>
<td>0.84</td>
<td>0.85</td>
<td>0.80</td>
<td>0.85</td>
<td>0.83</td>
<td>0.77</td>
<td>0.79</td>
<td>0.81</td>
</tr>
</tbody>
</table>

Sources: US SCF. Missing entries indicate net worth reported as 0. Entries with a proportion over the value of 1
indicates liquid debt holdings.

A.2.2 Comparison to US Flow of Funds

As also demonstrated in Kuhn and Ríos-Rull (2016), the aggregate measures in the SCF closely
match those available from an alternative source, the US Flow of Funds (FoF) (Table B100).
This is true both for the aggregate net household wealth, shown in Figure 8, and the housing
component of net wealth, shown in Figure 9.
A.3 Federal Reserve Asset Portfolio

Data for maturity of Federal Reserve Treasury holdings are taken from weekly statements produced by the Federal Reserve Bank of New York (FRBNY). The total value of the Federal Reserve System Open Market Account (SOMA) portfolio is over $4.2tn. This is comprised predominantly of US Treasury securities, worth $2.4tn (57.9% of the balance), with the bulk of remainder held as Mortgage Backed Securities (MBS), worth $1.8tn (41.9% of the balance). The SOMA account also contains a small quantity of federal agency securities, worth 0.2% of the total.

This paper predominately focusses on quantitative easing, with US Treasury holdings the most relevant assets. Maturity dates are given for each US Treasury held in the SOMA account. As the maturity date of MBS securities are uncertain, due to the possibility of mortgage prepayments, these are excluded from the maturity analysis presented here. These considerations are particularly relevant as nominal interest rates fall, as existing US mortgage holders may be tempted to prepay and re-mortgage at a lower nominal interest rate.

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## A.3.1 Additional Tables

### Table 18: SOMA Treasury Holdings Data

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<tr>
<th></th>
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<td>Par Value ($tn)</td>
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<td>0.4</td>
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<td>Remaining Maturity (years)</td>
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<td></td>
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<tr>
<td>Mean</td>
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<td>2.9</td>
<td>3.0</td>
<td>3.7</td>
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<td>6.2</td>
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<tr>
<td>Median</td>
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<td>5.4</td>
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<td>4.1</td>
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<td>Proportion of SOMA Balance Sheet with Remaining Maturity</td>
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<tr>
<td>&lt; 1 year</td>
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<td>55.8</td>
<td>54.5</td>
<td>47.9</td>
<td>24.1</td>
<td>11.8</td>
<td>8.4</td>
<td>7.7</td>
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<td>0.0</td>
<td>0.1</td>
<td>8.6</td>
<td>7.6</td>
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<tr>
<td>1-5 years</td>
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<td>31.9</td>
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<td>44.1</td>
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<td>20.1</td>
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<td>33.7</td>
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<td>28.9</td>
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<td>&gt; 10 years</td>
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<td>9.2</td>
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<td>10.4</td>
<td>18.8</td>
<td>16.7</td>
<td>13.7</td>
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<td>24.2</td>
<td>23.7</td>
<td>23.6</td>
</tr>
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</table>

Source: Federal Reserve Bank of New York. Data are taken on the first release of the given year, for 2003 the release on 09/07/2003 is used. Data uses US Treasury Bills, Notes and Bonds only.