Part IIA - Labour and Unemployment
Additional Notes

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Supervision 3: Expected Duration of Unemployment

In lectures we are told that the expected duration of unemployment is given as:

$$\mathbb{E}_t[D_u] = \frac{1}{f},$$

where f is the job finding rate. This note proves this statement.

In lectures we assumed that time was discrete. A person is either unemployed for 1 time period, 2 time periods, 3 time periods etc. Therefore, we may find the expected duration of unemployment by multiplying the length of every possible unemployment spell by the probability of being unemployed for that duration. This is simply the definition of an expected value for a discrete variable, for example:

$$\mathbb{E}_t[x] = \sum_{i=1}^{\infty} x_i p_i,$$

where the probability of event x_i arising is given by p_i.

For a given unemployed worker, the probabilities involved here are nice as they refer to the finding rate, f. As this is an independent variable, in any given time period (week, month, year etc...) the probability of a worker finding a job is f, while the probability that they remain unemployed is then $1 - f$.

Consider three cases:

1. The probability that an unemployed worker finds a job in one time period is simply the job finding rate, f.

2. The probability an unemployed worker finds a job after two time periods must be $f(1-f)$ as they failed to find a job after one time period, and find one after the second.

3. The probability an unemployed worker finds a job after three time periods must be $f(1-f)^2$ as they failed to find a job after time periods one and two, but find one after the third.

Hence, as stated in the lecture notes, that the expected duration of unemployment may be written as:

$$\mathbb{E}_t[D_u] = 1f + 2f(1-f) + 3f(1-f)^2 + \cdots = \frac{1}{f},$$

where D_u is the duration of unemployment and f is the job finding rate. To prove the second equality we use a trick. Notice that:

$$(1-f)\mathbb{E}_t[D_u] = 1f(1-f) + 2f(1-f)^2 + 3f(1-f)^3 + \cdots,$$

such that:

$$\mathbb{E}_t[D_u] - (1-f)\mathbb{E}_t[D_u] = f\mathbb{E}_t[D_u] = f + f(1-f) + f(1-f)^2 + f(1-f)^3 + \cdots,$$

Hence, $\mathbb{E}_t[D_u]$ may be rewritten as:

$$\mathbb{E}_t[D_u] = 1 + (1-f) + (1-f)^2 + (1-f)^3 + \cdots,$$

and using the mathematical result for the sum to infinity of a geometric series, $S_\infty = \sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}$, provided $r \neq 1$:

$$\mathbb{E}_t[D_u] = \frac{1}{1-(1-f)} = \frac{1}{f}.$$