Part IIA - Intertemporal Macroeconomics
Additional Notes

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Supervision 1: The Intertemporal Budget Constraint

An infinitely-lived consumer faces a series of period budget constraints:

$$\begin{aligned}
Y_1 + (1+r)B_0 &= C_1 + B_1, \\
Y_2 + (1+r)B_1 &= C_2 + B_2, \\
Y_3 + (1+r)B_2 &= C_3 + B_3, \\
&\cdots
\end{aligned}$$

where $C_t \geq 0$ denotes real consumption (in period t); $Y_t \geq 0$ denotes real income; B_t denotes period t saving and r denotes the real interest rate (which is assumed constant). This series of period budget constraints can be combined by eliminating saving in each period. For example, combining the first two equations:

$$\begin{aligned}
Y_2 + (1+r)B_1 &= C_2 + B_2, \\
Y_2 + (1+r)[Y_1 + (1+r)B_0 - C_1] &= C_2 + B_2, \\
Y_1 + (1+r)B_0 + \frac{Y_2}{(1+r)} &= C_1 + \frac{C_2}{1+r} + \frac{B_2}{1+r}.
\end{aligned}$$

Adding the next period budget constraint then gives:

$$(1+r)B_0 + Y_1 + \frac{Y_2}{(1+r)} + \frac{Y_3}{(1+r)^2} = C_1 + \frac{C_2}{1+r} + \frac{C_3}{(1+r)^2} + \frac{B_3}{(1+r)^2}.$$

This process can be continued until arriving at the condition:

$$(1+r)B_0 + \sum_{t=1}^{\infty} \frac{Y_t}{(1+r)^{t-1}} = \sum_{t=1}^{\infty} \frac{C_t}{(1+r)^{t-1}} + \lim_{t\to\infty} \frac{B_t}{(1+r)^{t-1}}.$$

One condition and one assumption are usually then made to generate the intertemporal budget constraint (IBC).

1. **Optimality** condition (also called the Transversality condition, TVC). In the limit, it will never be optimal for consumers to allow assets to grow without bound, such that:
$$\lim_{t\to\infty} \frac{B_t}{(1+r)^{t-1}} \leq 0.$$
2. **No-Ponzi** (No-Madoff) assumption. It is clearly optimal for the consumer to let debt grow without bound. This possibility is ruled out using the No-Ponzi condition.
$$\lim_{t\to\infty} \frac{b_t}{(1+r)^{t-1}} \geq 0.$$

Together these ensure the limit is set to 0, such that the intertemporal budget constraint is given as:

$$(1+r)B_0 + \sum_{t=1}^{\infty} \frac{Y_t}{(1+r)^{t-1}} = \sum_{t=1}^{\infty} \frac{C_t}{(1+r)^{t-1}}. \tag{IBC}$$

An additional assumption is sometimes used that consumers are born without assets $(1+r)B_0 = 0$. This would further simplify the IBC.

A Brief Note on Summations

The usual formula used above and in the lecture notes:

$$\sum_{t=1}^{\infty} \frac{X_t}{(1+r)^{t-1}} = X_1 + \frac{X_2}{(1+r)} + \frac{X_3}{(1+r)^2} + \cdots ,$$

associated with an index of summation: t, where time $\in [1, \infty)$.

An alternative formula may be found in many papers:

$$\sum_{s=t}^{\infty} \frac{X_s}{(1+r)^{s-t}} = X_t + \frac{X_{t+1}}{(1+r)} + \frac{X_{t+1}}{(1+r)^2} + \cdots ,$$

associated with an index of summation: s, where time $\in [t, \infty)$

Notice, by definition, the index of summation may not appear outside the summation operator. Therefore the second version is sometimes convenient as it allows the researcher to leave the current time period unspecified throughout analytical results.