

# Part IIB

## Supervision 9 - Revision I

### Growth and Business Cycles

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# This Class

- ▶ Your Questions.
- ▶ Faculty Essay 1: Migration and growth.
- ▶ Business Cycle Essay: 2018 Exam.
- ▶ Revision: Some guidance.

Your Questions.

## Question 1 - Growth: Spillovers Model

- ▶ Why is perpetual growth only achieved under the knowledge spillover model when  $\mu = 1 - \alpha$ , and not when  $\mu > 1 - \alpha$ .
- ▶ Recall growth rate of capital from that model:

$$g_k(t) = sA[k(t)]^{\alpha+\mu-1} - (\delta+n), \quad \Rightarrow \quad \frac{dg_k(t)}{dt} = (\alpha+\mu-1)g_k(t).$$

- ▶ Case 1:  $\alpha + \mu > 1$ : Explosive growth with no BGP as  $g_k(t) \rightarrow \infty$ .
- ▶ Case 2:  $\alpha + \mu < 1$ : Converge to BGP with  $g_k(t) = 0$ , as in Solow.
- ▶ Case 3:  $\alpha + \mu = 1$ : BGP with constant  $g_k(t) > 0$ .

## Question 2 - Growth: Lecture 5, Phase Diagram - Intuition

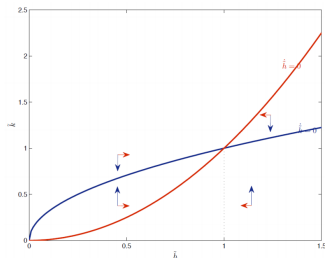
- ▶ Can you explain the human capital model phase diagram ?
- ▶ We are looking for the stationary point in the system:

$$\begin{aligned}\dot{\tilde{k}} &= s_K \tilde{k}^\alpha \tilde{h}^\gamma - (\delta + n + g)\tilde{k}, \\ \dot{\tilde{h}} &= s_H \tilde{k}^\alpha \tilde{h}^\gamma - (\delta + n + g)\tilde{h}.\end{aligned}$$

- ▶ We therefore trace out two lines:
  - ▶ Level of  $\tilde{k}$  and  $\tilde{h}$  for which  $\dot{\tilde{k}} = 0$  (concave).
  - ▶ Level of  $\tilde{k}$  and  $\tilde{h}$  for which  $\dot{\tilde{h}} = 0$  (convex).
- ▶ Their intersection is the steady state.

## Question 2 - Growth: Lecture 5, Phase Diagram - Graph

- ▶ For  $\tilde{k}$  above (below) the  $\dot{\tilde{k}} = 0$  line,  $g_{\tilde{k}} < 0$  ( $> 0$ ).
- ▶ We therefore we move towards the  $\dot{\tilde{k}} = 0$  line.
- ▶ For  $\tilde{h}$  left (right) of the  $\dot{\tilde{h}} = 0$  line,  $g_{\tilde{h}} > 0$  ( $< 0$ ).
- ▶ We therefore we move towards the  $\dot{\tilde{h}} = 0$  line.



Source: Tiago, Lecture 5.

## Question 3 - Growth: Lecture 7, Slide 15

- ▶ Is there a typo in the lectures here?
- ▶ Answer: **No**. But this is confusing.
- ▶ Slide considers three cases:  $\phi > 0$ ,  $\phi = 0$  and  $\phi < 0$ , but lecture then uses different boundaries  $\phi > 1$ ,  $\phi = 1$  and  $\phi < 1$ .
- ▶ This slide talks about the **rate of discovery** i.e.  $\dot{A}$ .
- ▶ But for BGP we consider the **growth rate**, i.e.  $\frac{\dot{A}}{A}$ .
- ▶ These are therefore consistent since:

$$\dot{A} = BL_A^\lambda A^\phi, \quad \Rightarrow \quad \frac{\partial \dot{A}}{\partial A} = \phi BL_A^\lambda A^{\phi-1},$$
$$g_k = \frac{\dot{A}}{A} = BL_A^\lambda A^{\phi-1}, \quad \Rightarrow \quad \frac{\partial g_k}{\partial A} = (\phi - 1)BL_A^\lambda A^{\phi-2}.$$

Question 4 - ADDITIONAL QUESTIONS WILL BE  
ADDED HERE



## Question 5 - Business Cycles: Utility

- ▶ What is the intuition behind  $\sigma$  in:

$$U(C_0, l_0) = \frac{C_0^{1-\sigma}}{1-\sigma} + \gamma \frac{(1-l_0)^{1-\sigma}}{1-\sigma}.$$

- ▶ Actually, to be well defined:

$$U(C_0, l_0) = \frac{C_0^{1-\sigma} - 1}{1-\sigma} + \gamma \frac{(1-l_0)^{1-\sigma} - 1}{1-\sigma}.$$

- ▶ Differentiate:

$$U'_C = C_0^{-\sigma} \quad U''_C = -\sigma C_0^{-\sigma-1}.$$

- ▶ Put together:

$$\text{Coef. of RRA} \equiv -\frac{U''_C}{U'_C} C = -\frac{-\sigma C_0^{-\sigma-1}}{C_0^{-\sigma}} C_0 = \sigma.$$

## Question 6 - Business Cycles: Investment

- ▶ RBC models have a single representative household. How does borrowing work?
- ▶ As always, the equilibrium condition for household saving is:

$$S(r) = I(r).$$

- ▶ Remember, this equilibrium condition simply determines the price,  $r$ .
- ▶ Without capital, we would simply write:

$$S(r) = 0.$$

## Question 7 - Business Cycles: Lecture 6, Slide 13 - Welfare

- ▶ What is meant by the asymmetric welfare effects in NK model.
- ▶ This is complicated! ... and usually ignored.
- ▶ Distortion 1: Imperfect competition ( $P$  too high,  $Y^n$  too low).
- ▶ Distortion 2: Sticky prices (sluggish response with  $Y_t \neq Y^n$ ).
- ▶ Output: actual,  $Y_t$ ; natural (flexible) level,  $Y^n$ ; and efficient (welfare maximising) level,  $Y^*$ .
- ▶ If  $Y^n < Y^*$ , a shock that moves  $Y_t > Y^n$ , moves economy closer to  $Y^*$ .

## Question 8 - Business Cycles: Shocks

- ▶ How do supply and demand shocks arise in the RBC and NK models?

- ▶ Supply shocks enter through productivity changes (TFP):

$$Y_t = A_t F(K_t, L_t).$$

- ▶ In NK model these may also enter via marginal costs shocks.

$$P_t = \frac{\eta}{\eta - 1} \frac{W_t}{A_t}.$$

- ▶ Why MC shocks in NK only?

- ▶ Demand shocks typically enter through the utility function:

$$U_t(C_t, \ell_t, \zeta_t) = \zeta_t(\log C_t + \ell_t),$$

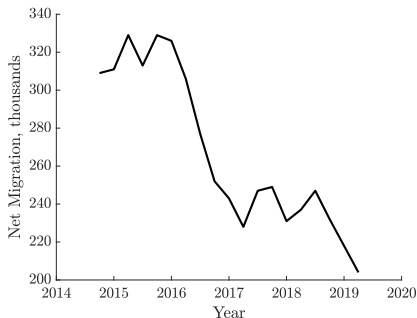
causing changes to Euler condition.

## Essay 1: Migration and Growth.

## Essay 1 - Set up

- ▶ *Figure 1 shows that net migration has been falling in the United Kingdom (UK) since Dec 2014. Critically assess the short-run and long-run impact of this slowdown of population growth on the UK economy.*

Figure 1: Net migration in the United Kingdom (unit: thousands): Dec 2014 to Sep 2019.



Source: Office for National Statistics.

## Essay 1 - Initial Steps

- ▶ Is this an AND or an OR question?
- ▶ Firstly set up a model, focus on growth literature.
- ▶ Discuss long run implications and the transition between steady states.
- ▶ RBC literature is another potential route. Essay likely to focus on labour supply elasticity and policy responses.

## Essay 1 - Continuous Time Solow Growth Model

- ▶ Production technology is represented by:

$$Y(t) = K(t)^\alpha L(t)^{1-\alpha}, \quad \text{with } \alpha \in (0, 1)$$

where  $Y(t)$  is output,  $K(t)$  is capital stock and  $L(t)$  is labour.

- ▶ Native labour force,  $L(t)$ , grows at constant rate  $n > 0$ , and new immigrants,  $M(t)$ , are a constant fraction of the labour force,  $m > 0$ , such that  $\frac{M(t)}{L(t)} = m$ , such that  $\frac{\dot{L}(t)}{L(t)} = n + m$ .
- ▶ Households save a fraction  $s \in (0, 1)$  of income. The economy is closed which implies that investment equals savings.
- ▶ Immigrants bring  $\psi$  units of capital with them, such that the law of motion for the capital stock is:

$$\dot{K}(t) = I(t) + \psi M(t) - \delta K(t).$$

where  $I(t)$  is investment and  $\delta > 0$  is the depreciation rate.



## Essay 1 - Find the Balanced Growth Path I

- ▶ The law of motion for capital is:

$$\dot{K}(t) = sK(t)^\alpha L(t)^{1-\alpha} + \psi M(t) - \delta K(t).$$

- ▶ The growth rate of  $K$  is then:

$$g_K \equiv \frac{\dot{K}(t)}{K(t)} = s \left( \frac{L(t)}{K(t)} \right)^{1-\alpha} + \psi m \frac{L(t)}{K(t)} - \delta.$$

so  $L(t)$  and  $K(t)$  grow at the same rate along a BGP.

- ▶ Define  $k \equiv \frac{K}{L}$  and write law of motion as:

$$\frac{\dot{K}(t)}{L(t)} = s \frac{K(t)^\alpha L(t)^{1-\alpha}}{L(t)} + \psi \frac{M(t)}{L(t)} - \delta \frac{K(t)}{L(t)},$$

$$\frac{\dot{K}(t)}{L(t)} = sk(t)^\alpha + \psi m - \delta k(t).$$

## Essay 1 - Find the Balanced Growth Path II

- ▶ Recall quotient rule states:

$$\dot{k}(t) \equiv \frac{\dot{K}(t)L(t) - \dot{L}(t)K(t)}{[L(t)]^2} = \frac{\dot{K}(t)}{L(t)} - (n + m)k(t).$$

- ▶ Use in above to give:

$$\dot{k}(t) = sk(t)^\alpha + \psi m - (\delta + n + m)k(t).$$

- ▶ Does the system converge to a BGP?

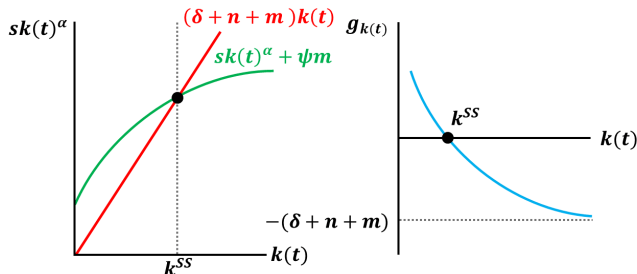
$$g_k \equiv \frac{\dot{k}(t)}{k(t)} = sk(t)^{\alpha-1} + \frac{\psi m}{k(t)} - (\delta + n + m),$$

with  $\lim_{k(t) \rightarrow \infty} g_k = -(\delta + n + m)$ , and  $\lim_{k(t) \rightarrow 0} g_k = \infty$ .

## Essay 1 - Graphical Solution

- ▶ Savings schedule higher than previously, by  $\psi m$ . Depreciation rate steeper,  $mk(t)$ .
- ▶ Growth rate remains downwards sloping. Looks similar.

Solow Model with Net Migration.



## Essay 1 - Return to Question

- ▶ Use framework to assess impact of a fall in  $m$ .
- ▶ On level of capital per worker. Set  $g_k = 0$  and implicitly differentiate. Rearrange to give:

$$\frac{dk^{SS}}{dm} = \frac{(\psi - k^{SS})k^{SS}}{(1 - \alpha)s(k^{SS})^\alpha + \psi m}.$$

- ▶ On growth rates.

$$\frac{dg_k}{dm} = \frac{\psi}{k(t)} - 1,$$

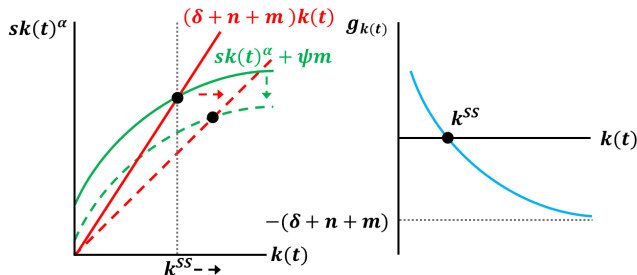
so a rotation about the point  $\psi$ .

- ▶ Take a stance that  $\psi < k^{SS}$  to give  $\frac{dk^{SS}}{dm} < 0$ .

# Essay 1 - Fall in Migration I

- ▶ Savings schedule **falls** as less capital brought to country, depreciation also **falls** as fewer people to share capital with.
- ▶ Net impact on  $k^{SS}$  is **unclear**. Here show an increase.

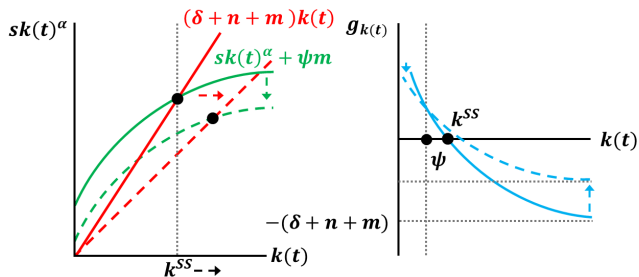
Solow Model with Net Migration.



# Essay 1 - Fall in Migration II

- ▶ Lower depreciation.
- ▶ Fewer people to share capital with.

Solow Model with Net Migration.



# Essay 1 - Transition Paths I

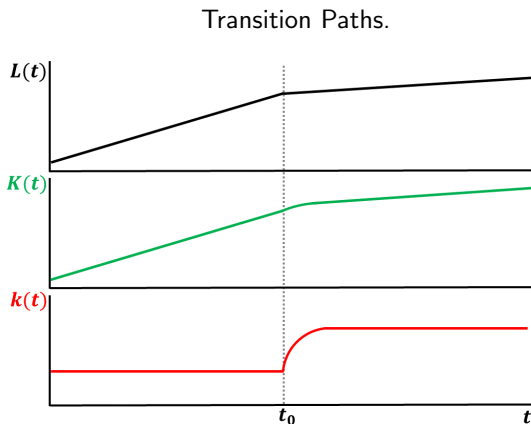
- ▶ Along the balanced growth path we have:

$$g_y = g_k = 0$$
$$g_K = g_L = g_Y = n + m.$$

- ▶ After  $m \downarrow$ ,  $K, L, Y$  will have a **lower** long run growth rate.
- ▶ Provided we begin at  $k^{SS}$  and  $k^{SS} > \psi$  then:
  - ▶ Long run  $k^{SS}$  **increases**.
  - ▶ Short run  $g_{k(t)}$  **increases**.

## Essay 1 - Transition Paths II

- ▶ “Kink” in path for  $L(t)$ , while  $K(t)$  adjusts more smoothly.
- ▶ Capital per worker initially increases rapidly.





# Essay 1 - Time Permitting, Something Special

- ▶ More detail on the literature.
- ▶ Look again at the data.

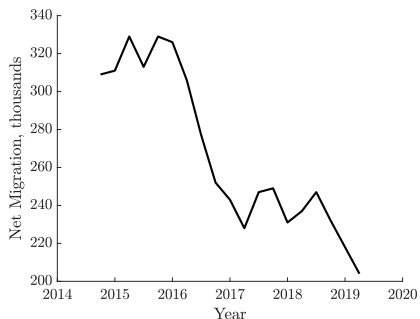
## Essay 1 - Literature

- ▶ A quick google search produces an **abundance** of literature.
  - ▶ Generally finds little impact of migration on wages or unemployment: Card (1990, 2005).
  - ▶ Also applies to the UK: Dustmann et al. (2005), Manacorda et al. (2012).
  - ▶ But impact may be highly concentrated along wage distribution: Dustmann et al. (2013).
- ▶ **Be careful here.** Top journals only. I knew where to look.
- ▶ Other techniques? Course references, dissertation knowledge...

## Essay 1 - Look Again at the Data I

- ▶ Take the original data series.
- ▶ Anything interesting? What could we do here?

Figure 1: Net migration in the United Kingdom (unit: thousands): Dec 2014 to Sep 2019.

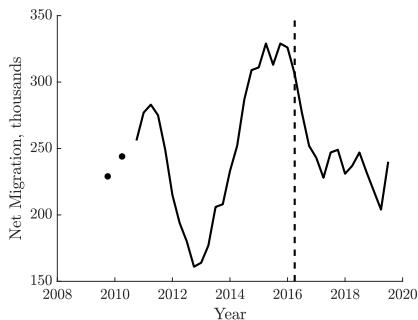


Source: Office for National Statistics.

## Essay 1 - Look Again at the Data II

- ▶ Extend it. What are the points of interest?
- ▶ **Be careful here:** time is precious.

Figure 1: Net migration in the United Kingdom (unit: thousands): Dec 2014 to Sep 2019.



Source: Office for National Statistics.

## Business Cycle Essay: 2018 Exam

## 2018 Exam - Set up

- ▶ Question 6: Critically assess the effectiveness of the New Keynesian model in explaining business cycle fluctuations and for policy analysis.
- ▶ Abridged answer (from study guide):
  - ▶ Present and explain NK model (assumptions and implications).
  - ▶ Present source of model shocks and propagation mechanisms.
  - ▶ Compare NK framework to RBC.

## 2018 Exam - Typical New Keynesian Model

- ▶ Households:
  - ▶ Decide consumption and leisure allocations.
  - ▶ **Key implication:** intertemporal decisions include “smoothing” motive.
- ▶ Two types of firms exist:
  - ▶ Intermediate goods firm operate with **imperfect competition**.
  - ▶ Final goods firms operate under **perfect competition**.
  - ▶ **Key implications:**  $P > MC$ , and prices could be “sticky”.
- ▶ Central bank follows rule to set  $M$  (or  $i$ ).

## 2018 Exam - Typical Shocks (Lecture 2, Slide 4)

- ▶ Shocks to: technology; monetary and fiscal policy; weather and natural disasters, (COVID?); political; expectations.
- ▶ Typically focus on technology and monetary. **Why?**
- ▶ Propagation mechanisms:
  - ▶ Intertemporal consumption/investment decision.
  - ▶ Labour decisions.
  - ▶ Financial mechanisms.



## 2018 Exam - Compare NK framework to RBC

- ▶ Key purpose of models is to explain business cycles.
- ▶ No role for money in RBC.
- ▶ No role for stabilisation policy in RBC.
- ▶ NK models couldn't explain 2007/8 crisis.
  - ▶ **Solution:** Add financial intermediation block.
- ▶ NK models can't explain COVID crisis.
  - ▶ **Solution:** Add SIR block.
- ▶ NK models still face monetary friction calibration problem.

# 2018 Exam - Time Permitting, Something Special

- ▶ More detail on the literature.
- ▶ More data.

## 2018 Exam - Literature

- ▶ **Everything** from the course.
- ▶ Most recent developments. Vasco's "Third Generation" HANK literature: Kaplan, Moll and Violante (2018).
- ▶ COVID literature: Kaplan, Moll, and Violante (2020), Guerrieri et al (2020), Eichenbaum et al (2020).
- ▶ Find Out More: Cambridge faculty COVID [webpage](#); CEPR policy blog, [VoxEu](#); Newspapers, e.g. [FT](#); online economics seminars, e.g. [VMACS](#).
- ▶ Again, **be very careful here**.

## 2018 Exam - Data

- ▶ Borrow from lecture notes.
- ▶ Do a HP-trend decomposition of output and show it.
- ▶ What are the key business cycle facts?
- ▶ How well do the RBC/NK models explain these.

## 2018 Exam - Data

- ▶ RBC (and also NK) models can replicate the relative volatility and autocorrelation of **quantities** surprisingly well.
- ▶ RBC models typically explain **prices** poorly.
  - ▶ The real interest rate is too persistent and not volatile enough.
  - ▶ Real wages and real interest rates are too procyclical.
  - ▶ .. and of course there's also something up with labour hours (which are not volatile or persistent enough).

Revision: Some Guidance.

## Revision: Ahead of Time

- ▶ Create document (including auto word count).
- ▶ Add all literature references in bibliography (group by topic).
- ▶ Your “known knowns”
  - ▶ Have you already done that question?
- ▶ Practise open book essays using previous exams.

## Revision: During the Exam

- ▶ Give both questions a chance.
- ▶ Quickly determine if question is an AND or an OR?
- ▶ Determine your answer approach:
  - ▶ Build a model.
  - ▶ Graphical explanation.
  - ▶ Critical essay.
- ▶ You **must** plan the essay (start with base essay, add detail).
- ▶ You **must** read over and check your full answer.



## Revision: What to Avoid

- ▶ Factual mistakes (including in literature).
- ▶ Not understanding the **main** point of question.
- ▶ Extensive use of lecture material (a false friend).
- ▶ Long derivations that go nowhere.

## Revision: Keep in Mind

- ▶ Like regular exam answers, the essay should be self-contained.
- ▶ Sufficient detail should be provided to understand it.
- ▶ The word limit of 2000 excludes mathematical expressions.
- ▶ The more precise and informative the essay is in addressing the question, the higher the mark.

## Final Thoughts

# Final Thoughts

- ▶ Question 1 similar to Q2 from the 2018 exam (and study aid).
- ▶ How to prepare for open book exams.
- ▶ COVID-19?
- ▶ Good luck!