EC421: International Economics

International Macroeconomics

Problem Set 1

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1 Two-Period Endowment Economy

Consider the two-country, two-period endowment economy discussed in Lecture 1 (OR, chapter 1). Assume the representative household in the Home country maximise:

$$U = \ln c_1 + \beta \ln c_2,$$

with $\beta \in (0, 1)$. The representative household in the foreign country has the same preferences over $\{c_1^*, c_2^*\}$.

(a) Let $\{y_1, y_2\}$ denote the Home endowment in the two periods. Solve for Home consumption in the first period.

Answer: Firstly we look to set up the maximisation problem for the representative household. The intertemporal budget constraint may be derived as:

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r},$$

which says that the present discounted value of consumption is equal to the present discounted value of endowment income. This is used to rewrite the household maximisation problem as:

$$\max_{c_1,c_2} \Big\{ \ln c_1 + \beta \ln c_2 \Big\} = \max_{c_1} \Big\{ \ln c_1 + \beta \ln[y_2 + (y_1 - c_1)(1+r)] \Big\}.$$

The first order condition of this problem is both necessary and sufficient for a solution, and rearrangement yields the standard Euler equation, discussed in lectures:

$$c_2 = \beta(1+r)c_1.$$

Plugging this into the the intertemporal budget constraint gives:

$$c_1 = \frac{1}{1+\beta}\left(y_1 + \frac{y_2}{1+r}\right),$$

where we have then shown that the value of consumption in the first period is given as a constant fraction of the present discounted value of income. This result is unsurprising given logarithmic utility.

(b) Solve for Home savings in the first period.

Answer: Define first-period savings as $s_1 = y_1 - c_1$. In this model, given $b_0 = i_1 = 0$ we have that $s_1 = b_1 = ca_1$. Using the result in part (a), we have:

$$s_1 = y_1 - c_1 = \frac{\beta y_1}{1+\beta} - \frac{y_2}{(1+\beta)(1+r)}.$$

This result makes intuitive sense and may be used to demonstrate the consumption smoothing behaviour in this model. An increase in y_1 will cause household saving to increase, as households seek to smooth income over time and only consume a constant fraction $\frac{1}{1+\beta}$ of current income. Similarly, an increase in future income, y_2 will results in a fall in household saving (or increased borrowing) as, facing higher income tomorrow, households spend more in period 1 in anticipation of future income gains.

(c) Let $\{y_1^*, y_2^*\}$ denote the Foreign endowment in the two period. Solve for the equilibrium world interest rate.

Answer: In this endowment economy, the global equilibrium requires:

$$s_1 + s_1^* = 0.$$

Because preferences in the Foreign country are the same preferences as in the Home country, s_1^* corresponds to s_1 after replacing for the Foreign endowment. Therefore:

$$s_1 + s_1^* = 0 \Rightarrow \frac{\beta y_1}{1+\beta} - \frac{y_2}{(1+\beta)(1+r)} + \frac{\beta y_1^*}{1+\beta} - \frac{y_2^*}{(1+\beta)(1+r)} = 0.$$

Solving for the real interest rate, we obtain:

$$1+r = \frac{1}{\beta}\left(\frac{y_2 + y_2^*}{y_1 + y_1^*}\right) = \frac{1}{\beta}\frac{y_2^W}{y_1^W}.$$

Where the final equality introduces the standard definitions of world income in both periods. Again, this is a very intuitive result.

(d) Show that the world interest rate must be between the real interest under autarky in the two countries.

Answer: We firstly define the shadow prices $1 + r^{Aut}$ and $1 + r^{Aut,*}$ using the condition $s_1 = s_1^* = 0$ as the real interest rates in the two countries under autarky. Using the expression for household saving derived in (b), combined with the autarky condition, we can obtain:

$$1 + r^{Aut} = \frac{1}{\beta}\frac{y_2}{y_1}.$$

Similarly, for the Foreign country, we have:

$$1 + r^{Aut,*} = \frac{1}{\beta}\frac{y_2^*}{y_1^*}.$$

To show the main result in the question we then have two options.

Option 1 (Direct): We substitute for both $1 + r^{Aut}$ and $1 + r^{Aut,*}$ into the equilibrium condition derived in (c) for the world real interest rate. We do this through a rearrangement to:

$$\beta y_1(1 + r^{Aut}) = y_2, \qquad \text{and} \qquad \beta y_1^*(1 + r^{Aut,*}) = y_2^*.$$

hence:

$$\begin{aligned}
1 + r &= \frac{1}{\beta}\left(\frac{y_2 + y_2^*}{y_1 + y_1^*}\right), \\
1 + r &= \frac{1}{\beta}\left(\frac{\beta y_1(1 + r^{Aut}) + \beta y_1^*(1 + r^{Aut,*})}{y_1 + y_1^*}\right), \\
1 + r &= \left(\frac{y_1}{y_1 + y_1^*}\right)(1 + r^{Aut}) + \left(\frac{y_1^*}{y_1 + y_1^*}\right)(1 + r^{Aut,*}),
\end{aligned}$$

which shows that the world real interest rate is a weighted average of the two autarky real interest rates. The weights are clearly both between 0 and 1, and sum to equal 1. The world real interest rate must therefore be between these two autarky rates.

Option 2 (Logic): In this setting we need to show that either $1 + r^{Aut} > 1 + r > 1 + r^{Aut,*}$ or $1+r^{Aut} < 1+r < 1+r^{Aut,*}$. The condition $1+r^{Aut} > 1+r$ corresponds to a restriction on the growth rate of the endowment in the two countries. We see this through the following chain of inequalities:

$$1 + r^{Aut} > 1 + r \quad \leftrightarrow \quad \frac{1}{\beta}\frac{y_2}{y_1} > \frac{1}{\beta}\left(\frac{y_2 + y_2^*}{y_1 + y_1^*}\right) \quad \leftrightarrow \quad y_2 y_1 + y_2 y_1^* > y_2 y_1 + y_2^* y_1 \quad \leftrightarrow \quad \frac{y_2}{y_1} > \frac{y_2^*}{y_1^*}.$$

The same condition is sufficient to satisfy the second part of the inequality $1 + r > 1 + r^{Aut,*}$.

Symmetrically, we can also show that the opposite inequality will result from the opposite initial conjecture ($1 + r^{Aut} < 1 + r < 1 + r^{Aut,*}$) and hence for all cases the world real interest rate must be between the two real interest rates under financial autarky.

(e) Show that the country with an autarky interest rate lower than the world interest rate will run a current account surplus in period 1.

Answer: Two methods are again available here.

Option 1 (Direct): We substitute for $1 + r^{Aut}$ into the equilibrium condition derived in (d) for household saving (here equal to the current account):

$$ca_1 = \frac{\beta y_1}{1+\beta} - \frac{y_2}{(1+\beta)(1+r)},$$
$$ca_1 = \frac{\beta y_1}{1+\beta} - \frac{\beta y_1(1+r^{Aut})}{(1+\beta)(1+r)},$$
$$ca_1 = \frac{\beta y_1}{1+\beta}\Big(1 - \frac{1+r^{Aut}}{1+r}\Big),$$

such that whenever $1 + r^{Aut} < 1 + r$ we have $ca_1 > 0$, as desired.

Option 2: Alternatively we may substitute the equilibrium condition for the world real interest rate directly into the equation for the current account.

$$ca_1 = \frac{\beta y_1}{1+\beta} - \frac{y_2}{(1+\beta)(1+r)},$$
$$ca_1 = \frac{\beta y_1}{1+\beta} - \frac{y_2}{(1+\beta)\frac{1}{\beta}\left(\frac{y_2+y_2^*}{y_1+y_1^*}\right)},$$
$$ca_1 = \frac{\beta}{1+\beta}\left(y_1 - \frac{y_2}{\left(\frac{y_2+y_2^*}{y_1+y_1^*}\right)}\right),$$
$$ca_1 = \frac{\beta}{1+\beta}\left(\frac{y_1y_2 + y_1y_2^* - y_2y_1 - y_2y_1^*}{y_2+y_2^*}\right),$$
$$ca_1 = \frac{\beta}{1+\beta}\left(\frac{y_1y_2^* - y_2y_1^*}{y_2+y_2^*}\right),$$

such that we observe $ca_1 > 0$ when:

$$\frac{y_1}{y_2} > \frac{y_1^*}{y_2^*},$$

which from part (d) we identify as the condition for $1 + r^{Aut} < 1 + r$.

(f) What is the effect of an increase in the growth rate of Foreign output on Home's welfare?

Answer: Using the Euler equation, we can rewrite welfare of the Home representative household as:

$$U = \beta \ln \beta + (1+\beta) \ln c_1 + \beta \ln(1+r).$$

The effect of an increase of the growth rate of Foreign output on the Home representative household's welfare is:

$$\frac{\partial U}{\partial (y_2^*/y_1^*)} = \frac{1+\beta}{c_1} \frac{\partial c_1}{\partial (y_2^*/y_1^*)} + \frac{\beta}{1+r} \frac{\partial (1+r)}{\partial (y_2^*/y_1^*)}$$

First notice that, from part (c), we can write the world interest rate as:

$$1+r = \frac{1}{\beta} \frac{\frac{y_2}{y_1^*} + \frac{y_2^*}{y_1^*}}{1+y_1}.$$

From this expression, it is clear that the world interest rate is increasing in the growth rate of the endowment in the Foreign country:

$$\frac{\partial (1+r)}{\partial (y_2^*/y_1^*)} > 0.$$

From the solution for c_1, we can write:

$$\frac{\partial c_1}{\partial (y_2^*/y_1^*)} = -\frac{y_2}{(1+\beta)(1+r)^2} \frac{\partial (1+r)}{\partial (y_2^*/y_1^*)}.$$

Plugging back into the derivative of welfare with respect to the growth rate of Foreign endowment and simplifying, we obtain:

$$\frac{\partial U}{\partial (y_2^*/y_1^*)} = \frac{1}{1+r} \left[\beta - \frac{y_2}{(1+r)c_1} \right] \frac{\partial (1+r)}{\partial (y_2^*/y_1^*)}.$$

Using again the Euler equation, the last expression becomes:

$$\frac{\partial U}{\partial (y_2^*/y_1^*)} = \frac{\beta}{1+r} \left(\frac{c_2 - y_2}{c_2} \right) \frac{\partial (1+r)}{\partial (y_2^*/y_1^*)}.$$

Therefore, the sign of the effect of a change in the growth rate of the Foreign endowment depends on whether the Home country consumes more than its endowment in the second period or not. Using the solution for consumption in the first period and the Euler equation, we can write:

$$c_2 - y_2 = \frac{\beta(1+r)}{1+\beta} y_1 + \frac{\beta}{1+\beta} y_2 - y_2.$$

After substituting for the world real interest rate and rearranging, we can arrive at:

$$c_2 - y_2 = \frac{1}{1+\beta} \frac{y_1 y_1^*}{y_1 + y_1^*} \left(\frac{y_2^*}{y_1^*} - \frac{y_2}{y_1} \right).$$

Therefore, the effect of a change in the growth rate of Foreign output on the Home household's welfare will depend on the initial position. Recall that $y_2^*/y_1^* > y_2/y_1 \Rightarrow r^{Aut} < r < r^{Aut,*}$. If the Home country starts with an autarky interest rate lower than the Foreign one, an increase of Foreign output growth will be beneficial, and vice versa. Since a low autarky interest rate reflects a desire to save, an increase in the Foreign country's endowment represents a positive development because the Foreign country would like to borrow (to smooth its expected income growth by anticipating consumption) exactly when the Home country is willing to lend. This may also be shown diagrammatically.

2 Persistent Output Growth

Consider the stochastic infinite horizon small open endowment economy model discussed in Lecture 1 (OR, chapter 2; SGU, chapter 2). Assume utility is quadratic and that $\beta(1+r) = 1$.

(a) Show that the one-period change in consumption equals the present discounted value of the change in expected future output levels:

$$c_{t+1} - c_t = \frac{r}{1+r} \sum_{j=0}^{\infty} \frac{(\mathbb{E}_{t+1} - \mathbb{E}_t) y_{t+1+j}}{(1+r)^j}.$$

Answer: The intertemporal budget constraint gives:

$$\sum_{j=0}^{\infty} \frac{\mathbb{E}_t c_{t+j}}{(1+r)^j} = (1+r) b_{t-1} + \sum_{j=0}^{\infty} \frac{\mathbb{E}_t y_{t+j}}{(1+r)^j}.$$

With quadratic utility, the Euler equation implies that consumption is a random walk:

$$\mathbb{E}_t c_{t+1} = c_t.$$

Replacing into the intertemporal budget constraint gives:

$$c_t = r b_{t-1} + \frac{r}{1+r} \sum_{j=0}^{\infty} \frac{\mathbb{E}_t y_{t+j}}{(1+r)^j}.$$

Updating one period, we get:

$$c_{t+1} = r b_t + \frac{r}{1+r} \sum_{j=0}^{\infty} \frac{\mathbb{E}_{t+1} y_{t+1+j}}{(1+r)^j}.$$

Taking expectations of the last expression yields:

$$\mathbb{E}_t c_{t+1} = r b_t + \frac{r}{1+r} \sum_{j=0}^{\infty} \frac{\mathbb{E}_t y_{t+1+j}}{(1+r)^j},$$

where we used the law of iterated expectations in the infinite sum. Finally, taking the difference between the last two expressions gives the result:

$$c_{t+1} - c_t = \frac{r}{1+r} \sum_{j=0}^{\infty} \frac{(\mathbb{E}_{t+1} - \mathbb{E}_t) y_{t+1+j}}{(1+r)^j},$$

where we have used again the random walk result from the Euler equation.

(b) Suppose output follows an AR(1) process in first differences

$$y_{t+1} - y_t = \rho(y_t - y_{t-1}) + \epsilon_{t+1},$$

where $\rho \in (0,1)$ and $\epsilon \sim i.i.d.\ \mathcal{N}(0, \sigma_\epsilon^2)$. Show that $\forall j > 0$:

$$(\mathbb{E}_{t+1} - \mathbb{E}_t)y_{t+j} = \frac{1-\rho^j}{1-\rho}\epsilon_{t+1}.$$

Answer: Starting from the law of motion for output growth, forward substitution gives:

$$y_{t+j} - y_{t+j-1} = \rho^j(y_t - y_{t-1}) + \rho^{j-1}\epsilon_{t+1} + ... + \epsilon_{t+j}.$$

Taking expectations at $t+1$ yields:

$$\mathbb{E}_{t+1}(y_{t+j} - y_{t+j-1}) = \rho^j(y_t - y_{t-1}) + \rho^{j-1}\epsilon_{t+1},$$

while taking expectations at t gives:

$$\mathbb{E}_t(y_{t+j} - y_{t+j-1}) = \rho^j(y_t - y_{t-1}).$$

Taking the difference between the two, therefore, leads to:

$$\mathbb{E}_{t+1}(y_{t+j} - y_{t+j-1}) - \mathbb{E}_t(y_{t+j} - y_{t+j-1}) = \rho^{j-1}\epsilon_{t+1}.$$

We can rearrange the left-hand side to obtain a process for revision in expectations of output:

$$(\mathbb{E}_{t+1} - \mathbb{E}_t)y_{t+j} = (\mathbb{E}_{t+1} - \mathbb{E}_t)y_{t+j-1} + \rho^{j-1}\epsilon_{t+1}.$$

We can work backward until y_t, for which $(\mathbb{E}_{t+1} - \mathbb{E}_t)y_t = 0$, to obtain:

$$(\mathbb{E}_{t+1} - \mathbb{E}_t)y_{t+j} = (1 + \rho + ... + \rho^{j-1})\epsilon_{t+1} = \sum_{s=0}^{j-1}\rho^s\epsilon_{t+1} = \frac{1-\rho^j}{1-\rho}\epsilon_{t+1}.$$

(c) Derive an expression for consumption growth as a function of output innovations $y_{t+1} - \mathbb{E}_t y_{t+1}$.

Answer: Rewrite consumption growth from part (a) as:

$$c_{t+1} - c_t = \frac{r}{1+r}\sum_{j=1}^{\infty}\frac{(\mathbb{E}_{t+1} - \mathbb{E}_t)y_{t+j}}{(1+r)^{j-1}}.$$

Substitute for the result obtained in part (b):

$$c_{t+1} - c_t = \frac{r}{1+r} \sum_{j=1}^{\infty} \frac{1}{(1+r)^{j-1}} \frac{1-\rho^j}{1-\rho} \epsilon_{t+1} = \frac{r}{1+r} \frac{\epsilon_{t+1}}{1-\rho} \sum_{j=1}^{\infty} \frac{1-\rho^j}{(1+r)^{j-1}}.$$

We can split the sum as:

$$c_{t+1} - c_t = \frac{r}{1+r} \frac{\epsilon_{t+1}}{1-\rho} \left[\sum_{j=1}^{\infty} \frac{1}{(1+r)^{j-1}} - \sum_{j=1}^{\infty} \frac{\rho^j}{(1+r)^{j-1}} \right] = \frac{r}{1+r} \frac{\epsilon_{t+1}}{1-\rho} \left[\sum_{j=0}^{\infty} \frac{1}{(1+r)^j} - \rho \sum_{j=0}^{\infty} \left(\frac{\rho}{1+r} \right)^j \right].$$

From the last expression, using the formula for the geometric series, we obtain:

$$c_{t+1} - c_t = \frac{1+r}{1+r-\rho} \epsilon_{t+1} = \frac{1+r}{1+r-\rho} (y_{t+1} - \mathbb{E}_t y_{t+1}),$$

where the second equality comes from the process for output growth.

(d) What is the implication for the innovations in consumption as a function of the innovations in output?

Answer: Recall that the innovations in consumption correspond to consumption growth because of the random walk result. Therefore, from the result in the previous point, we can see that the innovations in consumption are more volatile than the innovations of output (since $\rho < 1$).

(e) What is the response of the current account to an innovation in output?

Answer: The current account is:

$$ca_t = rb_{t-1} + y_t - c_t.$$

Substituting the process for output and the solution for consumption, we can write:

$$ca_t = rb_{t-1} + y_{t-1} + \rho(y_{t-1} - y_{t-2}) - c_{t-1} + \left(1 - \frac{1+r}{1+r-\rho}\right) \epsilon_t.$$

Rearranging the last expression, we have:

$$\Delta ca_t = r\Delta b_{t-1} + \rho \Delta y_{t-1} - \frac{\rho}{1+r-\rho} \epsilon_t,$$

where Δ is the first-difference operator (that is, for any variable x_t, $\Delta x_t \equiv x_t - x_{t-1}$). Therefore, a positive innovation to current income leads to a deterioration of the current account.[1] This result is the opposite of what we concluded in Lecture 1 with persistent but stationary output shocks.

[1] Notice that the shock has no effect on the variables dated $t-1$.

3 Open Production Economy with Specialised Imports

Consider a small open economy existing for two periods, 1 and 2, populated by many identical agents with preferences:

$$\ln c_1 + \beta \ln c_2,$$

At time 1, the residents are endowed with an exogenous amount of y_1 of output. They can invest their savings either in foreign assets, b_1, which yield the world interest rate, $1 + r$, or in domestic projects, i_1, which yield output in period 2, Y_2, with decreasing returns:

$$Y_2 = A i_1^{\alpha}$$

The initial net foreign wealth is zero ($b_0 = 0$), so that b_1 coincides with the current account (i.e. $b_1 - b_0 = b_1 < 0$ represents the current account deficit). Investing in project i_1 requires a specialised imported input, at the price p units of output. Hence it costs $p \cdot i_1$ units of output (=consumption).

(a) Use the production function and budget constraints, construct the intertemporal production possibility frontier (IPPF) of available consumption bundles, under financial autarky.

Answer: The IPPF is defined as the set of intertemporal consumption options $\{c_1; c_2\}$ which may be obtained by varying investment. This is done under Financial Autarky, which infers these options must be obtained without the use of bonds.

The budget constraints under financial autarky:

$$c_1 = y_1 - p i_1,$$
$$c_2 = A i_1^{\alpha}.$$

Together these infer an IPPF:

$$c_2 = A\left[\frac{y_1 - c_1}{p}\right]^{\alpha}.$$

(b) Graph the IPPF and show some of its properties.

Answer: The IPPF has a negative slope as:

$$\frac{\partial c_2}{\partial c_1} = -\frac{\alpha A}{p}\left[\frac{y_1 - c_1}{p}\right]^{\alpha - 1} < 0,$$

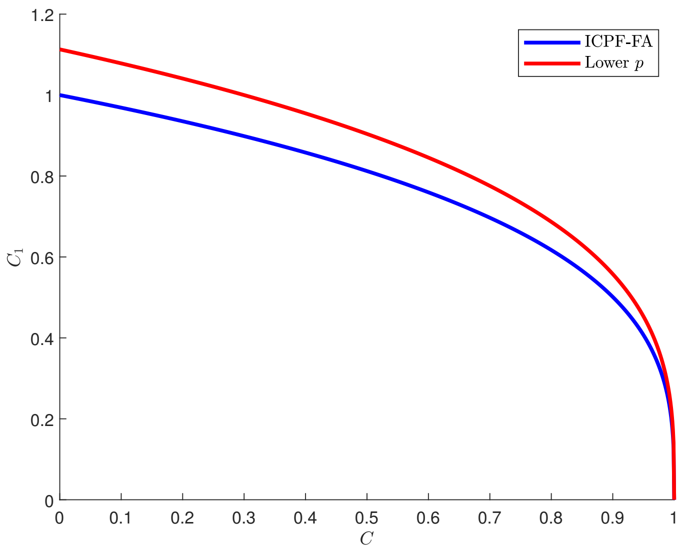
and is drawn in Figure 1.

(c) How does the IPPF change if the price of the imported input, p, falls? Explain.

Answer: Shifts outwards.

$$\frac{\partial c_2}{\partial c_1} = -\frac{\alpha A}{p}\left[\frac{y_1 - c_1}{p}\right]^{\alpha - 1} < 0,$$

Figure 1: IPPF



Notes: $Y = p_0 = 1$, $\alpha = 0.6$, $p_1 = 0.7$.

and is also drawn in Figure 1. We find that c_2 is decreasing in p at all levels of c_1. Intuitively, the higher the price of the imported input, the higher the amount of current consumption the country has to give up to finance the investment required to sustain a given target of c_2. The point on the x-axis does not move as this represents full investment (so a change in the relative price of investment has no impact), while the intersection with the y-axis moves by the largest amount (as this point represents a situation with no consumption in period 1 and full investment).

(d) Write down the budget constraint of the country in the two periods (including b_1) and the problem of the representative consumer. Derive the first order conditions characterising the optimal consumption and investment decisions.

Answer: The period budget constraints are:

$$c_1 = y_1 - pi_1 - b_1,$$
$$c_2 = Ai_1^{\alpha} + (1+r)b_1.$$

such that the representative consumer solves:

$$\max_{i_1, b_1} \Big\{ \ln(y_1 - pi_1 - b_1) + \beta \ln(Ai_1^{\alpha} + (1+r)b_1) \Big\},$$

and there are two first order conditions of the problem:

$$-\frac{p}{c_1}+\beta\frac{\alpha A i_1^{\alpha-1}}{c_2}=0, \qquad \rightarrow \qquad c_2=\frac{\beta\alpha A i_1^{\alpha-1}}{p}c_1$$
$$-\frac{1}{c_1}+\beta\frac{1+r}{c_2}=0 \qquad \rightarrow \qquad c_2=\beta(1+r)c_1.$$

We may combine these first order condition to obtain:

$$\underbrace{\frac{c_2}{\beta c_1}}_{MRS}=1+r=\underbrace{\frac{\alpha A i_1^{\alpha-1}}{p}}_{MRT},$$

which clearly shows that the optimality point says that the marginal rate of transformation of consumption between periods 1 and 2 must be set equal (or the rate of return on financial versus physical investment must be equal). This represents a portfolio allocation issue. The MRT is then equal to the MRS of consumption i.e. how much consumption in each period is valued by the household.

(e) Define and derive the interest rate under financial autarky, r^{Aut}. Is r^{Aut} increasing/decreasing in p? And in A?

Answer: r^{Aut} is the interest rate at which it is optimal to consume the financial autarky bundle even if trade in the international financial markets is possible.

An equilibrium must be both optimal and feasible. Using the first order conditions, optimality will require:

$$c_2=\frac{\beta\alpha A i_1^{\alpha-1}}{p}c_1,$$
$$(1+r)=\frac{\alpha A i_1^{\alpha-1}}{p}.$$

such that we link consumption decisions with investment, and the investment decision with the level of the real interest rate. Using the budget constraints under financial autarky, feasibility will require:

$$c_1^{Aut}=y_1-p i_1^{Aut},$$
$$c_2^{Aut}=A i_1^{Aut,\alpha}.$$

such that we have 4 equations and 4 unknowns. As every equation includes an expression for investment, i_1, once this is found in terms of exogenous variables all else will follow.

$$
\begin{aligned}
i^{Aut} &= \frac{\alpha\beta}{1+\alpha\beta}\frac{y_1}{p}, \\
(1+r^{Aut}) &= \frac{\alpha A}{p}\Big(\frac{\alpha\beta}{1+\alpha\beta}\frac{y_1}{p}\Big)^{\alpha-1} = \alpha A\Big(\frac{\alpha\beta}{1+\alpha\beta}y_1\Big)^{\alpha-1}p^{-\alpha}.
\end{aligned}
$$

We then observe that $1+r^{Aut}$ is decreasing in p and increasing in A as we have:

$$
\begin{aligned}
\frac{\partial r^{Aut}}{\partial p} &= -\alpha^2 A\Big(\frac{\alpha\beta}{1+\alpha\beta}y_1\Big)^{\alpha-1}p^{-\alpha-1} < 0, \\
\frac{\partial r^{Aut}}{\partial A} &= \alpha\Big(\frac{\alpha\beta}{1+\alpha\beta}y_1\Big)^{\alpha-1}p^{-\alpha} > 0.
\end{aligned}
$$

Intuitively, a higher price of imported input is akin to lower growth, as it makes future consumption more valuable at any level of current consumption.

(f) Assume that preferences and technology are such that $1+r^{Aut} > 1+r$. Is the current account going to be positive or negative? Explain.

Answer: Residents in countries with fundamentals such that $r^{Aut} > r$ will value present consumption over future consumption more than residents abroad. These countries will tend to import present consumption and thus run a CA deficit. The best answers here will describe this graphically.

(g) Derive the consumption and investment plans if agents can trade the international bond.

Answer: In a trading equilibrium we face a similar set of four conditions. However, now r is now given, rather than endogenous, and b_1 is to be found.

$$
\begin{aligned}
c_1 &= y_1 - pi_1 - b_1, \\
c_2 &= Ai_1^{\alpha} + (1+r)b_1, \\
c_2 &= \frac{\beta\alpha A i_1^{\alpha-1}}{p}c_1, \\
r &= \frac{\alpha A i_1^{\alpha-1}}{p}.
\end{aligned}
$$

It will again be easiest to eliminate for investment first, since all equations depend on i_1. Once this is found all else will follow, and leads to:

$$\tilde{i}_1 = \Big(\frac{\alpha A}{p(1+r)}\Big)^{\frac{1}{1-\alpha}},$$
$$\tilde{c}_1 = \frac{1}{1+\beta}\Big[y_1 - p\tilde{i}_1 + \frac{A\tilde{i}_1^{\alpha}}{1+r}\Big].$$

with consumption a constant fraction of the net present value of wealth, after investment decisions (an implication of the log functional form of utility).