

# EC421: International Economics

## International Macroeconomics

### Problem Set 5

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## 1 New Open Economy Macroeconomics

In the workhorse two-country symmetric model of the New Open Economy Macroeconomics, with one period preset-prices analysed during Lecture 5, the expected utility of the representative agent in the Home country is:

$$\mathbb{E}_{t-1}[U(C_t, \ell_t)] = \mathbb{E}_{t-1}[\ln(C_t) - \kappa \ell_t] = \mathbb{E}_{t-1} \left[ \ln \frac{\mu_t}{P_{H,t}^{1/2} P_{F,t}^{1/2}} \right] - \kappa \bar{\ell},$$

where  $\mu_t$  is defined as the monetary stance ( $\mu_t \uparrow$  is an expansion) under the control of the monetary authorities.

Assume Producer Currency Pricing (PCP). Let  $\mathcal{E}_t$  denote the nominal exchange rate, with  $\mathcal{E}_t = \frac{\mu_t}{\mu_t^*}$ . The variables  $Z_t$  and  $Z_t^*$  denote productivity at home and abroad,  $mkp$  is the constant equilibrium markup charged by firms. Foreign variables are starred. Using the fact that in equilibrium  $\kappa W_t = \kappa P_t C_t = \kappa \mu_t$ , the optimality preset prices charged by domestic and foreign producers are:

$$P_{H,t} = mkp \cdot \mathbb{E}_{t-1} \left[ \frac{\kappa \mu_t}{Z_t} \right],$$
$$P_{F,t}^* = mkp \cdot \mathbb{E}_{t-1} \left[ \frac{\kappa \mu_t^*}{Z_t^*} \right]$$

so that import prices in the home country are  $P_{F,t} = \mathcal{E}_t P_{F,t}^*$ .

Analogous expressions characterise the foreign country, that is:

$$\begin{aligned}\mathbb{E}_{t-1}[U(C_t^*, \ell_t^*)] &= \mathbb{E}_{t-1} \left[ \ln \frac{\mu_t^*}{P_{H,t}^{*,1/2} P_{F,t}^{*,1/2}} \right] - \kappa \bar{\ell}, \\ \mathcal{E}_t P_{H,t}^* &= m k p \cdot \mathbb{E}_{t-1} \left[ \frac{\kappa \mu_t}{Z_t} \right], \\ P_{F,t}^* &= m k p \cdot \mathbb{E}_{t-1} \left[ \frac{\kappa \mu_t^*}{Z_t^*} \right].\end{aligned}$$

Assume that the only source of uncertainty consists of *iid* shocks to productivity,  $Z_t$  and  $Z_t^*$ .

- (a) Taking the monetary stance in the foreign country,  $\mu_t^*$ , as given, write the policy problem of the home monetary authorities, assuming that these are welfare maximising and can commit.

**Answer:** The policy problem is:

$$\begin{aligned}\max_{\mu_t} \mathbb{E}_{t-1}[U(C_t^*, \ell_t^*)] &= \max_{\mu_t} \mathbb{E}_{t-1} \left[ \ln \mu_t - \frac{1}{2} \ln P_{H,t} - \frac{1}{2} \ln P_{F,t} \right], \\ &= \max_{\mu_t} \mathbb{E}_{t-1} \left[ \ln \mu_t - \frac{1}{2} \ln m k p - \frac{1}{2} \ln \mathbb{E}_{t-1} \left[ \frac{\kappa \mu_t}{Z_t} \right] - \frac{1}{2} \ln \mathcal{E}_t - \frac{1}{2} \ln m k p - \frac{1}{2} \ln \mathbb{E}_{t-1} \left[ \frac{\kappa \mu_t^*}{Z_t^*} \right] \right], \\ &= \max_{\mu_t} \mathbb{E}_{t-1} \left[ \frac{1}{2} \ln \mu_t - \frac{1}{2} \ln m k p - \frac{1}{2} \ln \mathbb{E}_{t-1} \left[ \frac{\kappa \mu_t}{Z_t} \right] + \frac{1}{2} \ln \mu_t^* - \frac{1}{2} \ln m k p - \frac{1}{2} \ln \mathbb{E}_{t-1} \left[ \frac{\kappa \mu_t^*}{Z_t^*} \right] \right].\end{aligned}$$

where we substitute for the functional forms of both prices and the nominal exchange rate in reformulating the maximisation problem.

- (b) Derive the optimal policy

**Answer:** The first order condition with respect to  $\mu_t$  is:

$$\frac{1}{2} \frac{1}{\mu_t} - \frac{1}{2} \frac{\frac{\kappa}{Z_t}}{\mathbb{E}_{t-1} \left[ \frac{\kappa \mu_t}{Z_t} \right]} = 0,$$

such that by rearrangement we have:

$$\frac{\kappa \mu_t}{Z_t} = \mathbb{E}_{t-1} \left[ \frac{\kappa \mu_t}{Z_t} \right],$$

which may be solved with a policy function of the form:

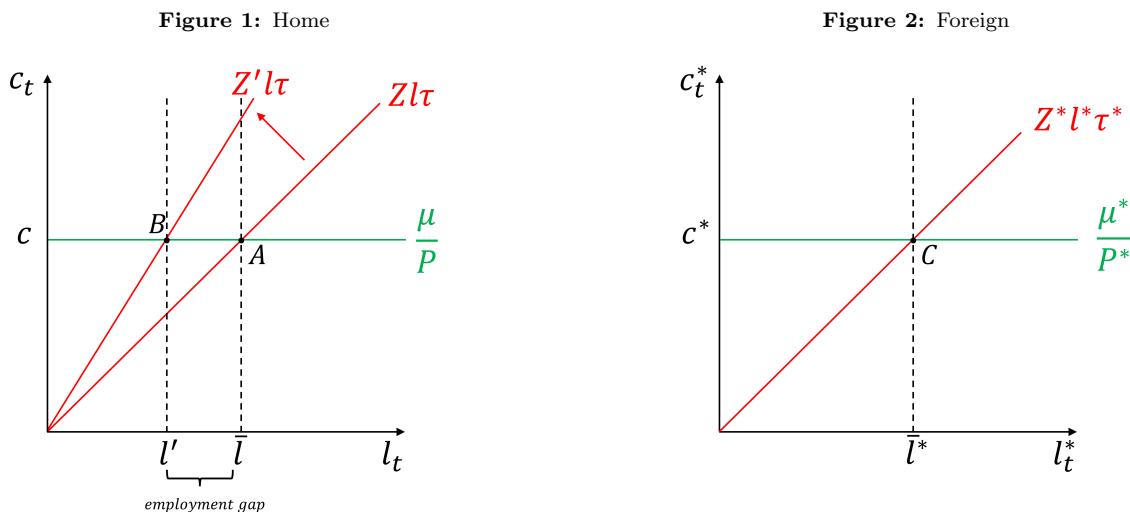
$$\mu_t = \Gamma Z_t,$$

showing that optimal policy will consist of matching an increase in productivity with a monetary expansion. Notice that this policy rule is “inward looking” as monetary policymakers in the Home

country respond only to domestic productivity shocks. The policy function is the same as in a closed economy.

- (c) Carefully explain the effect on both the Home and Foreign economies of an increase in  $Z_t$  with no-policy response, and under the optimal policy.

**Answer:** With rigid prices, holding  $\mu_t$  and  $\mu_t^*$  constant,  $C_t$  and  $\mathcal{E}_t$  do not move  $Z_t \uparrow$  will then lead to an output gap in the home country, as shown in Figure 1. Absent a policy response there will be no spillovers of a productivity change to the Foreign country, as shown in Figure 2.



Source: Corsetti and Pesenti (2007).

In contrast, with optimal policy we have that:

$$\mu_t = \Gamma Z_t,$$

and the home policy maker responds with expansionary monetary policy to a positive productivity shock. In the case with PCP this causes a terms of trade deterioration in the Home country and an improvement for Foreign. We have positive spillovers of monetary policy stabilisation, such that gains from the productivity shock are shared between countries. The scenario for the Home country is shown in Figure 3, and for the Foreign country in Figure 4.

In terms of international prices, in response to an exogenous monetary policy easing,  $\mu_t \uparrow$ , the nominal exchange rate will depreciate  $\mathcal{E}_t = \frac{\mu_t}{\mu_t^*} \uparrow$ , while the (consumption-output) terms of trade will deteriorate,  $\tau_t \downarrow$ .

Under “sticky” prices with PCP pricing, monetary policy easing in the Home country will lead to expenditure switching effects in the short-run. A monetary policy easing which leads to an exchange rate depreciation will cause the price of Foreign goods to increase for the Home country, and the

price of Home goods to fall in the Foreign country. In both cases the relative price of Foreign goods increases, such that households in both countries demand more Home goods and fewer Foreign. Consumption shifts in favour of the cheaper product.

Figure 3: Home

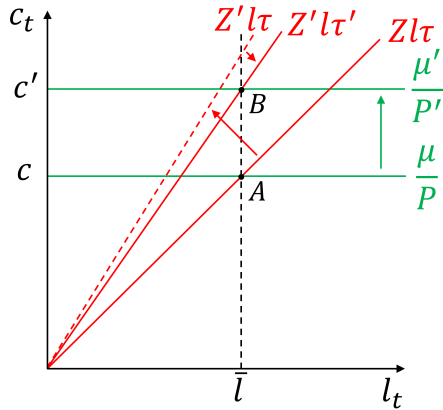
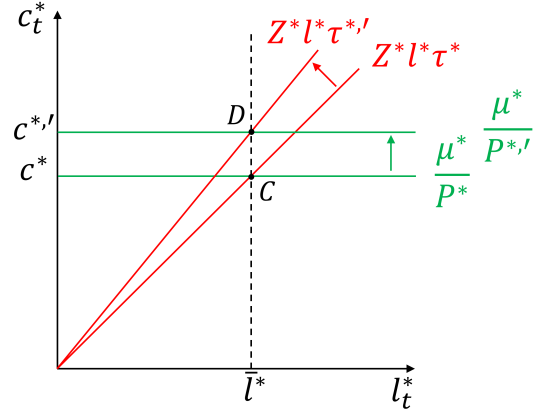


Figure 4: Foreign



Source: Corsetti and Pesenti (2007).

(d) Would the optimal policy under cooperation be different? Explain?

**Answer:** The policy problem under cooperation will be the solution to the equation:

$$\max_{\mu_t, \mu_t^*} \mathbb{E}_{t-1} \left[ \frac{U(c_t, l_t) + U(c_t^*, l_t^*)}{2} \right].$$

Under PCP the uncooperative and cooperative solutions coincide. Notice that the policy prescription for PCP did not depends on the level of  $\mu^*$ . There are no gains to international policy coordination, therefore, and the global welfare maximising solutions is for every country to simply “keep their own house in order”. The monopoly effects of the home country terms of trade are internalised by the home country.

Note: In principle there is no reason why the policy rules must coincide but, in practise, they do for this case.

## 2 Re-Assessing Optimal Stabilisation

While the New Open Economy Macroeconomic models discussed in Lecture 5 suggests the gains from going from Nash optimal policy to a coordinated international policy may be small, we are also able to assess the gains from stabilisation altogether.

- (a) Assume the same PCP setting as in Question 1, and posit the monetary policymakers in the Home country obey the policy function:

$$\mu_t = Z_t,$$

such that policymakers respond to productivity shocks one-to-one. Assume that Foreign monetary stance,  $\mu_t^*$  is fixed. Calculate Welfare,  $\mathcal{W}_t|_{\mu_t=Z_t}$ , for the Home country in this case.

**Answer:** Home welfare is given as:

$$\mathcal{W}_t = \mathbb{E}_{t-1}[U(C_t, \ell_t)] = \mathbb{E}_{t-1}[\ln(C_t) - \kappa \ell_t] = \mathbb{E}_{t-1} \left[ \ln \frac{\mu_t}{P_{H,t}^{1/2} P_{F,t}^{1/2}} \right] - \kappa \bar{\ell}.$$

Under PCP pricing we may plug in price indices, and use the LOOP, to give:

$$\mathcal{W}_t = \mathbb{E}_{t-1}[\ln \mu_t] - \frac{1}{2} \mathbb{E}_{t-1} \ln P_{H,t} - \frac{1}{2} \mathbb{E}_{t-1} \ln P_{F,t} - \kappa \bar{\ell},$$

$$\mathcal{W}_t = \mathbb{E}_{t-1}[\ln \mu_t] - \frac{1}{2} \mathbb{E}_{t-1} \ln P_{H,t} - \frac{1}{2} \mathbb{E}_{t-1} \ln \mathcal{E}_t - \frac{1}{2} \mathbb{E}_{t-1} \ln P_{F,t}^* - \kappa \bar{\ell},$$

$$\mathcal{W}_t = \mathbb{E}_{t-1}[\ln \mu_t] - \frac{1}{2} \mathbb{E}_{t-1} \ln P_{H,t} - \frac{1}{2} \mathbb{E}_{t-1} \ln \mu_t + \frac{1}{2} \mathbb{E}_{t-1} \ln \mu_t^* - \frac{1}{2} \mathbb{E}_{t-1} \ln P_{F,t}^* - \kappa \bar{\ell},$$

$$\mathcal{W}_t = \frac{1}{2} \mathbb{E}_{t-1}[\ln \mu_t] - \frac{1}{2} \mathbb{E}_{t-1} \ln P_{H,t} + \frac{1}{2} \mathbb{E}_{t-1} \ln \mu_t^* - \frac{1}{2} \mathbb{E}_{t-1} \ln P_{F,t}^* - \kappa \bar{\ell},$$

$$\mathcal{W}_t = \frac{1}{2} \mathbb{E}_{t-1}[\ln \mu_t] - \mathbb{E}_{t-1}[\ln m k p \cdot \kappa] - \frac{1}{2} \mathbb{E}_{t-1}[\ln \mathbb{E}_{t-1}[\mu_t/Z_t]] + \frac{1}{2} \mathbb{E}_{t-1} \ln \mu_t^* - \frac{1}{2} \mathbb{E}_{t-1}[\ln \mathbb{E}_{t-1}[\mu_t^*/Z_t^*]] - \kappa \bar{\ell},$$

$$\mathcal{W}_t = \frac{1}{2} \mathbb{E}_{t-1}[\ln \mu_t] - \mathbb{E}_{t-1}[\ln m k p \cdot \kappa] - \frac{1}{2} \ln \mathbb{E}_{t-1}[\mu_t/Z_t] + \frac{1}{2} \mathbb{E}_{t-1} \ln \mu_t^* - \frac{1}{2} \ln \mathbb{E}_{t-1}[\mu_t^*/Z_t^*] - \kappa \bar{\ell}.$$

Finally, use the specific reaction function given in the question,  $\mu_t = Z_t$ . Firms know this policy function before setting prices. Welfare is therefore given as:

$$\mathcal{W}_t|_{\mu_t=Z_t} = \frac{1}{2} \mathbb{E}_{t-1}[\ln Z_t] - \mathbb{E}_{t-1}[\ln m k p \cdot \kappa] + \frac{1}{2} \mathbb{E}_{t-1} \ln \mu_t^* - \frac{1}{2} \ln \mathbb{E}_{t-1}[\mu_t^*/Z_t^*] - \kappa \bar{\ell}.$$

- (b) Instead, now assume monetary policymakers in the Home country obey the policy function:

$$\mu_t = 1,$$

such that policy is invariant to productivity shocks. Calculate welfare of the Home country,  $\mathcal{W}_t|_{\mu_t=1}$ ,

and hence compute the welfare gains from full stabilisation.

**Answer:** Using the result from part (a) Home welfare becomes:

$$\mathcal{W}_t|_{\mu_t=1} = -\mathbb{E}_{t-1}[\ln mkp \cdot \kappa] - \frac{1}{2} \ln \mathbb{E}_{t-1}[1/Z_t] + \frac{1}{2} \mathbb{E}_{t-1} \ln \mu_t^* - \frac{1}{2} \ln \mathbb{E}_{t-1}[\mu_t^*/Z_t^*] - \kappa \bar{\ell},$$

such that the gains from stabilisation may be written directly as:

$$\mathcal{W}_t|_{\mu_t=Z_t} - \mathcal{W}_t|_{\mu_t=1} = \frac{1}{2} \left( \mathbb{E}_{t-1}[\ln Z_t] + \ln \mathbb{E}_{t-1}[1/Z_t] \right),$$

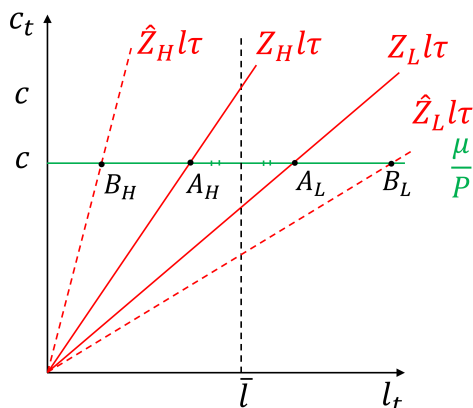
- (c) Show, graphically, how an increase in the variance of productivity shocks may lead to higher gains from stabilisation policy.

**Answer:** Conjecture the simple case of a mean preserving spread of two possible productivity states, arising with the same probability,  $Z_H$  and  $Z_L$ . An increase in the variance of  $Z$  translates into a greater distance between both  $B_H$  and  $B_L$  from the natural rate allocation,  $\bar{\ell}$ , as shown in Figure 5 (compared to  $A_H$  and  $A_L$ ). This infers, in the absence of stabilisation policies, the employment (and hence output) gaps will be larger.

In the event of a positive productivity shock (absent stabilisation policy), employment falls below the natural rate. With nominal expenditure fixed, prices are “too high” in this case. In contrast, with a low productivity realisation employment increases beyond the natural rate. Prices are “too low” in this case.

Under full stabilisation we close the output gap in both cases. Thus the difference (and therefore gains) from stabilisation are of second order importance, as these are related to the variance of the productivity shock.

**Figure 5:** Mean Preserving Spread



Source: Corsetti and Pesenti (2007).

- (d) In Lecture 5 we claim that optimal monetary policy under discretion may introduce an incentive for policymaker to deviate ex post. Using a graphical illustration, explain why.

**Answer:** Given prices and a monetary policy rule, monetary policymakers who act under discretion to maximise utility will always have an incentive to deviate ex post, after observing the shocks.

Note that the optimal allocation in the economy arises where the indifference curves are tangent to the production constraint. The scenario for a closed-economy is shown in Figure 6, which highlights how the monetary authority would want to eliminate the monopoly distortion after observing a shock, rather than remaining at the natural rate.

In an open economy setting the terms of trade represent an additional consideration. The case for PCP pricing is shown in Figure 7. In this case, the additional expansion by monetary policymakers (aimed at eliminating the monopoly distortions) is harmful to the home economy, as the terms of trade deteriorate. The optimal solution under discretion in this case therefore balances the extent to which household gain from additional production, with the loss of international competitiveness associated with loose monetary policy. Clearly, this limits the extent to which domestic policymakers enact policy, ex post. Consider also the case of LCP pricing.

Figure 6: Closed Economy

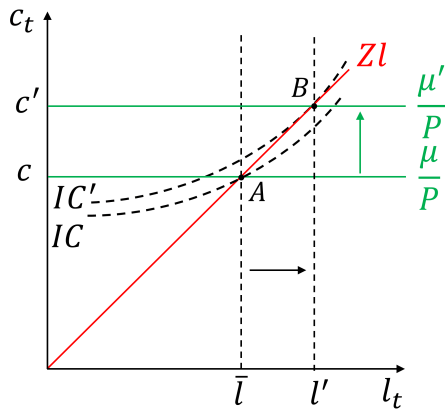
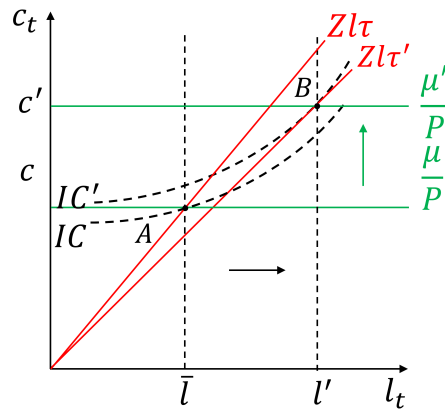


Figure 7: Open Economy (PCP)



Source: Corsetti and Pesenti (2007).

## References

Corsetti, G. and P. Pesenti (2007). The Simple Geometry of Transmission and Stabilization in Closed and Open Economies. In R. Clarida and F. Giavazzi (Eds.), *NBER International Seminar on Macroeconomics*, Number Chapter 2, Chapter 2, pp. University of Chicago Press. Chicago, IL: University of Chicago Press.