EC421: International Economics

International Macroeconomics

Problem Set 4

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1 The Hodrick-Prescott Filter

Consider the HP filter as outlined during the lecture, for the set of data $\{y_t\}_{t=1}^T$. The loss function is summarised by:

$$L = \sum_{t=1}^{T}(y_t - y_t^T)^2 + \lambda \sum_{t=2}^{T-1}[(y_{t+1}^T - y_t^T) - (y_t^T - y_{t-1}^T)]^2,$$

where $\lambda \geq 0$ and our objective is to extract the trend $\{y_t^T\}_{t=1}^T$ from the original data.

(a) Provide an intuitive explanation for this loss function, and how for how it aims to separate data from trend.

(b) Find an analytical solution to this loss function for the case when $\lambda = 0$, and provide an intuitive explanation for this result.

(c) Find an analytical solution to this loss function for the case when $\lambda =\rightarrow \infty$, and provide an intuitive explanation for this result.

(d) Download quarterly, seasonally-adjusted, US real GDP data for 1947Q1 - 2018Q1.[1] Using a computer programme of your choice (MatLab, Stata, EViews...) firstly, take the logarithm of the raw data, and then calculate the value of HP-filtered real GDP. Plot both series, and the accompanying cyclical data (as a percentage of trend).

[1] This may be found either from the St. Louis Federal Reserve data services at https://fred.stlouisfed.org/ or the Bureau of Economic Analysis at https://www.bea.gov/.

(e) Repeat this exercise (using the same quarterly data) for the HP filter but now setting $\lambda = 100$. Comment on the cyclical properties of the data.

2 International Real Business Cycles in a Small Open Economy

This question follows the structure in Mendoza (1991) and Chapter 4 of Schmitt-Grohé and Uribe (2017). Consider the planner problem for a small open economy. Preferences for a representative household are defined over consumption, c_t, and labour supply, ℓ_t given by:

$$U_t(c_t, \ell_t) = \mathbb{E}_t\Big[\sum_{s=t}^{\infty} \beta^{s-t} \frac{(c_s - \frac{\ell_s^{\omega}}{\omega})^{1-\gamma} - 1}{1-\gamma}\Big],$$

where $\gamma > 0$ and $\omega > 0$. The representative household maximises utility subject to a period-t budget constraint, give as:

$$b_t + c_t + i_t + \Phi(k_t - k_{t-1}) = y_t + (1 + r_{t-1})b_{t-1},$$

where b_t denotes the representative agent's net holdings of international financial assets and the function $\Phi(x)$ captures capital stock, k_t, adjustment costs with a functional form given as $\Phi(x) = \frac{\phi}{2}(x)^2$. r represents the real rate of interest on net foreign assets, and is explained further, below. Output, y_t, is produced by means of a linearly homogeneous productions functions that takes the capital and labour as inputs such that:

$$y_t = z_t k_{t-1}^{\alpha} \ell_t^{1-\alpha}.$$

where z_t is an exogenous technology process. The law of motion for domestic capital is given as:

$$k_t = (1 - \delta)k_{t-1} + i_t,$$

and, finally, the exogenous technology process is assumed to follow an AR(1) structure in logs with:

$$\ln(z_t) = \rho \ln(z_{t-1}) + \varepsilon_t,$$

where $\varepsilon_t \sim N(0, \sigma_{\varepsilon}^2)$.

(a) Set up the problem facing the social planner in this model. What are the choice variables? What are the endogenous and exogenous state variables? Do you have enough of each? Eliminate the constraints to reduce the problem as much as possible.

(b) Firstly, calculate the optimality conditions for the planner's problem. Then, use these equilibrium relationships to eliminate the Lagrange multiplier, λ_t from the problem.

(c) We will solve the problem in two steps. Firstly, we consider the steady state (stationary point) of

Table 1: Parameter values

β	=	0.96	Discount factor
γ	=	2	Coefficient of relative risk aversion
δ	=	0.1	Depreciation rate
α	=	0.32	Capital income share
r^*	=	0.04	World real interest rate
$\bar{b}$	=	0.7442	NFA without interest rate premium
ψ_1	=	$\bar{b}/100$	Debt sensitivity of interest rate
ϕ	=	0.028	Capital adjustment cost parameter
ω	=	1.455	IES of labour supply, $(\frac{1}{\omega-1})$
ρ	=	0.42	AR(1) technology parameter
σ_ε^2	=	0.0129^2	Technology shock variance

the system. We consider a non-stochastic stationary point, such that $z_t = \bar{z}$. Re-write the optimality conditions from part (c) which will determine the steady state.

(d) A well-documented problem in this setting is that the Euler equation will not help in determining any of the endogenous variables, and therefore the system has an infinite number of solutions. To side-step this issue, we will assume an endogenous real interest rate for the small open economy which depends critically on the level of net foreign assets chosen by households. However this dependence of the real interest rate on the level of net foreign assets is not internalised by households, who simply observe that:

$$r_t = r^* + \psi_1(\exp^{\bar{b}-b_t} - 1),$$

where the real interest rate now includes a risk premium term which is a convex function with respect to external debt. The real interest rate for the small open economy therefore becomes an endogenous variable, our number of unknown variables increases by 1 as does our number of equilibrium conditions. Show how this relationship, combined with the Euler equation can may help determine an endogenous variable in the steady state.

(e) We now solve for the steady state explicitly. Use Matlab, along with its in-built fsolve function, to write a compute programme to solve for the steady state. Use the parameters as given in Table 1. *Hint: Rewrite the equilibrium conditions into a vector of equations which may then be set equal to 0.*

(f) **Extension:** Using Dynare, write a computer program to solve the model.[2] Again, Table 1 provides

[2]Dynare is downloadable for free at http://www.dynare.org/. It works with Matlab and Octave on Windows, Mac, and Linux platforms. See the Dynare webpage for more details, including the manual and user guide that will explain in details its functionalities.

the parameter values of the model.

[Hint: In the Dynare folder 'examples', you will find a number of models which you should be able to run easily on your computers. As a starting point, you can manipulate one of these to accommodate the differences with the model in this exercise. Notice the file 'bkk.mod' will solve the model in Backus, Kehoe and Kydland (1992), but note that this may not be the most simplistic model to manipulate.]

References

Mendoza, E. G. (1991). Real Business Cycles in a Small Open Economy. *American Economic Review 81*(4), 797–818.

Schmitt-Grohé, S. and M. Uribe (2017). *Open Economy Macroeconomics.* Princeton, NJ: Princeton University Press.