EC421: International Economics

International Macroeconomics

Problem Set 3

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1 Consumption Bundles and Price Indexes

Consider an economy inhabited by a representative households who consumes tradable and non-tradable goods according to an overall consumption index:

$$C \equiv \left[\gamma^{\frac{1}{\theta}} C_T^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} C_N^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}},$$

with $\gamma \in (0,1)$ and $\theta > 0$. In turn, the tradable consumption index is a CES aggregator of tradable goods produced in country H and F:

$$C_T \equiv \left[\mu^{\frac{1}{\eta}} C_H^{\frac{\eta-1}{\eta}} + (1-\mu)^{\frac{1}{\eta}} C_F^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

with $\mu \in (0,1)$ and $\eta > 0$.

(a) Define the Home price of tradable goods produced in country $j = \{H, F\}$ as P_j and the price of non-tradable goods as P_N. Find the price indexes P_T and P.

Answer: We need to find the price P_T that solves:

$$P_T C_T = \min_{C_H, C_F} P_H C_H + P_F C_F,$$

subject to:

$$C_T \equiv \left[\mu^{\frac{1}{\eta}} C_H^{\frac{\eta-1}{\eta}} + (1-\mu)^{\frac{1}{\eta}} C_F^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} = \bar{C}_T.$$

Let the multiplier on the quantity constraint be λ and form the Lagrangian:

$$\mathcal{L} = P_H C_H + P_F C_F - \lambda \left\{ \left[\mu^{\frac{1}{\eta}} C_H^{\frac{\eta-1}{\eta}} + (1-\mu)^{\frac{1}{\eta}} C_F^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} - \bar{C}_T \right\}.$$

The first order conditions (FOCs) are:

$$P_H - \lambda \left(\frac{\eta}{\eta-1} \right) \left[\mu^{\frac{1}{\eta}} C_H^{\frac{\eta-1}{\eta}} + (1-\mu)^{\frac{1}{\eta}} C_F^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}-1} \mu^{\frac{1}{\eta}} \left(\frac{\eta-1}{\eta} \right) C_H^{\frac{\eta-1}{\eta}-1} = 0,$$

and:

$$P_F - \lambda \left(\frac{\eta}{\eta-1} \right) \left[\mu^{\frac{1}{\eta}} C_H^{\frac{\eta-1}{\eta}} + (1-\mu)^{\frac{1}{\eta}} C_F^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}-1} (1-\mu)^{\frac{1}{\eta}} \left(\frac{\eta-1}{\eta} \right) C_F^{\frac{\eta-1}{\eta}-1} = 0.$$

Note that:

$$\left[\mu^{\frac{1}{\eta}} C_H^{\frac{\eta-1}{\eta}} + (1-\mu)^{\frac{1}{\eta}} C_F^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}-1} = C_T^{\frac{1}{\eta}}.$$

We can then rewrite the FOCs as:

$$P_H = \lambda \mu^{\frac{1}{\eta}} \left(\frac{C_H}{C_T} \right)^{-\frac{1}{\eta}} \Rightarrow C_H = \mu \left(\frac{P_H}{\lambda} \right)^{-\eta} C_T, \tag{1}$$

$$P_F = \lambda (1-\mu)^{\frac{1}{\eta}} \left(\frac{C_F}{C_T} \right)^{-\frac{1}{\eta}} \Rightarrow C_F = (1-\mu) \left(\frac{P_F}{\lambda} \right)^{-\eta} C_T. \tag{2}$$

We can then substitute the last two results into the expression for the aggregator to obtain:

$$C_T = \left[\mu \left(\frac{P_H}{\lambda} \right)^{1-\eta} C_T^{\frac{\eta-1}{\eta}} + (1-\mu) \left(\frac{P_F}{\lambda} \right)^{1-\eta} C_T^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}.$$

If we simplify C_T, we can solve for λ and obtain:

$$\lambda = \left[\mu P_H^{1-\eta} + (1-\mu) P_F^{1-\eta} \right]^{\frac{1}{1-\eta}}. \tag{3}$$

Now, let's go back to (1) and (2) and multiply both sides of the demand for good $i = \{H, F\}$ by its respective price to get:

$$P_H C_H = \mu \lambda^{\eta} P_H^{1-\eta} C_T,$$

and:

$$P_F C_F = (1-\mu)\, \lambda^{\eta} P_F^{1-\eta} C_T.$$

Adding the last to expressions yields:

$$P_H C_H + P_F C_F = P_T C_T = \lambda^{\eta} \left[\mu P_H^{1-\eta} + (1-\mu) P_F^{1-\eta} \right] C_T.$$

From (3), we note that the expression in brackets is equal to $\lambda^{1-\eta}$. Simplifying for C_T, we then find that:

$$P_T = \lambda. \tag{4}$$

Therefore, the price index P_T that minimizes spending on the two varieties is:

$$P_T = \left[\mu P_H^{1-\eta} + (1-\mu) P_F^{1-\eta} \right]^{\frac{1}{1-\eta}}. \tag{5}$$

Similarly, you can show that:

$$P = \left[\gamma P_T^{1-\theta} + (1-\gamma) P_N^{1-\theta} \right]^{\frac{1}{1-\theta}}. \tag{6}$$

(b) Consider a steady state in which relative prices equal one. What is the interpretation of μ and γ?

Answer: We can substitute for the Lagrange multiplier from (4) into the demand functions (1) and (2) to obtain:

$$C_H = \mu \left(\frac{P_H}{P_T} \right)^{-\eta} C_T \tag{7}$$

$$C_F = (1-\mu) \left(\frac{P_F}{P_T} \right)^{-\eta} C_T. \tag{8}$$

In a steady state in which relative prices are equal to one, we obtain:

$$\frac{C_H}{C_T} = \mu, \qquad \frac{C_F}{C_T} = 1 - \mu.$$

Therefore, μ represents the steady state share of tradable consumption devoted to Home goods. Similarly, γ is the tradable share of total consumption.

(c) Show that the elasticity of substitution between Home and Foreign tradables is constant. What is it equal to?

Answer: If we take the ratio of (7) and (8), we get:

$$\frac{C_H}{C_F} = \left(\frac{\mu}{1-\mu} \right) \left(\frac{P_H}{P_F} \right)^{-\eta}.$$

The elasticity of substitution between the two goods is given by:

$$\frac{\partial \ln (C_H/C_F)}{\partial \ln (P_H/P_F)} = -\eta,$$

which is constant. Note that sometimes the elasticity of substitution is expressed relative to the inverse of the relative price (i.e P_F/P_H) which is defined (in macro) as the the terms of trade.

(d) Show that:

$$\lim_{\eta \to 1} C_{Tt} = \frac{C_{Ht}^{\mu} C_{Ft}^{1-\mu}}{\mu^{\mu}(1-\mu)^{1-\mu}}$$

[*Hint: Transform the aggregator so that you can apply de L'Hospital rule*].

Answer: In this answer we will use two "tricks" to show the required result. In the first we will use L'Hospital's rule, as suggested by the hint.

First, note that L'Hospital's rule states that the limit of a ratio may be rewritten as the limit of the derivatives of that ratio, provided these exist. Specifically:

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)},$$

provided both $f'(x)$ and $g'(x)$ exist in the limit. This result has useful applications when the initial ratio, $\frac{f(x)}{g(x)}$ is not well specified in the limit.

To apply L'Hospital's rule to the problem at hand, we take logs of the consumption aggregator to form:

$$\ln C_T = \frac{\ln \left[\mu^{\frac{1}{\eta}} C_H^{\frac{\eta-1}{\eta}} + (1-\mu)^{\frac{1}{\eta}} C_F^{\frac{\eta-1}{\eta}} \right]}{(\eta-1)/\eta},$$

and then define:

$$f(\eta) \equiv \ln \left[\mu^{\frac{1}{\eta}} C_H^{\frac{\eta-1}{\eta}} + (1-\mu)^{\frac{1}{\eta}} C_F^{\frac{\eta-1}{\eta}} \right],$$
$$g(\eta) \equiv \frac{\eta-1}{\eta}.$$

so that the problem may be rewritten as:

$$\lim_{\eta \to 1} \ln C_T = \frac{f(\eta)}{g(\eta)} = \frac{0}{0},$$

which is clearly not well defined in the limit. We thus look to invoke L'Hospital's rule, by differentiating the functions $f(\eta)$ and $g(\eta)$.

The derivative of the denominator, $g(\eta)$ with respect to η is simply $1/\eta^2$, so that its limit for $\eta \to 1$ is equal to one, which provides us with hope that the ratio $\frac{f'(x)}{g'(x)}$ will be well defined. For the numerator, we use a second "trick".

In the second "trick" we note the following equality:

$$a^{h(x)} = e^{\ln a^{h(x)}} = e^{h(x)\ln a},$$

such that:

$$\frac{\partial a^{h(x)}}{\partial x} = \ln a \cdot h'(x) e^{h(x)\ln a} = \ln a \cdot h'(x) e^{\ln a^{h(x)}} = \ln a \cdot h'(x) \cdot a^{h(x)}.$$

We are interested in computing the differential of the numerator in the L'Hospital equation, $f'(\eta)$, where:

$$f(\eta) \equiv \ln \left[\mu^{\frac{1}{\eta}} C_H^{\frac{\eta-1}{\eta}} + (1-\mu)^{\frac{1}{\eta}} C_F^{\frac{\eta-1}{\eta}} \right].$$

Let us define:

$$j(\eta) \equiv \mu^{\frac{1}{\eta}} C_H^{\frac{\eta-1}{\eta}} + (1-\mu)^{\frac{1}{\eta}} C_F^{\frac{\eta-1}{\eta}},$$

such that, by using the derivative of logarithms, we know:

$$f'(\eta) = \frac{j'(\eta)}{j(\eta)}.$$

Next, to invoke the second "trick", re-write $j(\eta)$ as:

$$j(\eta) \equiv e^{\frac{1}{\eta}\ln \mu} e^{\frac{\eta-1}{\eta}\ln C_H} + e^{\frac{1}{\eta}\ln(1-\mu)} e^{\frac{\eta-1}{\eta}\ln C_F},$$

and notice that there are four terms involving η. The chain rule of differentiation will therefore be used to compute:

$$\begin{aligned}\frac{\partial j(\eta)}{\partial \eta} \equiv j'(\eta) = & -\frac{\ln \mu \cdot \mu^{\frac{1}{\eta}} e^{\frac{\eta-1}{\eta}\ln C_H}}{\eta^2} + \frac{e^{\frac{1}{\eta}\ln \mu} \ln C_H \cdot C_H^{\frac{\eta-1}{\eta}}}{\eta^2} \\ & -\frac{\ln(1-\mu)\cdot(1-\mu)^{\frac{1}{\eta}} e^{\frac{\eta-1}{\eta}\ln C_F}}{\eta^2} + \frac{e^{\frac{1}{\eta}\ln(1-\mu)} \ln C_F \cdot C_F^{\frac{\eta-1}{\eta}}}{\eta^2},\end{aligned}$$

which may be further rewritten as:

$$\begin{aligned} j'(\eta) &= \frac{1}{\eta^2}\Big[-\ln \mu \cdot \mu^{\frac{1}{\eta}} \cdot C_H^{\frac{\eta-1}{\eta}} + \mu^{\frac{1}{\eta}} \cdot \ln C_H \cdot C_H^{\frac{\eta-1}{\eta}} - \ln(1-\mu)\cdot(1-\mu)^{\frac{1}{\eta}} \cdot C_F^{\frac{\eta-1}{\eta}} + (1-\mu)^{\frac{1}{\eta}} \cdot \ln C_F \cdot C_F^{\frac{\eta-1}{\eta}} \Big], \\ &= \frac{1}{\eta^2}\Big[\mu^{\frac{1}{\eta}} \cdot C_H^{\frac{\eta-1}{\eta}} \ln\Big(\frac{C_H}{\mu}\Big) + (1-\mu)^{\frac{1}{\eta}} \cdot C_F^{\frac{\eta-1}{\eta}} \ln\Big(\frac{C_F}{1-\mu}\Big)\Big], \end{aligned}$$

Combining results, we therefore observe:

$$f'(\eta) = \frac{j'(\eta)}{j(\eta)} = \frac{\frac{1}{\eta^2}\left[\mu^{\frac{1}{\eta}} C_H^{\frac{\eta-1}{\eta}} \ln\left(\frac{C_H}{\mu}\right) + (1-\mu)^{\frac{1}{\eta}} C_F^{\frac{\eta-1}{\eta}} \ln\left(\frac{C_F}{1-\mu}\right)\right]}{\left[\mu^{\frac{1}{\eta}} C_{Ht}^{\frac{\eta-1}{\eta}} + (1-\mu)^{\frac{1}{\eta}} C_{Ft}^{\frac{\eta-1}{\eta}}\right]},$$

so that:

$$\lim_{\eta\to 1} = \frac{f'(\eta)}{g'(\eta)} = \mu \ln\left(\frac{C_H}{\mu}\right) + (1-\mu)\ln\left(\frac{C_F}{1-\mu}\right),$$

and existence is verified. Overall we then have that:

$$\lim_{\eta\to 1} \ln C_T = \mu \ln\left(\frac{C_H}{\mu}\right) + (1-\mu)\ln\left(\frac{C_F}{1-\mu}\right),$$

Such that, by taking the exponent of the last expression, we can conclude that:

$$\lim_{\eta\to 1} C_T = \frac{C_H^{\mu} C_F^{1-\mu}}{\mu^{\mu}(1-\mu)^{1-\mu}}.$$

(e) Assume that for the Foreign country $\gamma^* = \gamma$, $\theta^* = \theta$ and $\eta^* = \eta$ but $\mu^* \neq \mu$. Further assume that the law of one price holds for individual tradable goods ($P_j = P_j^*$). Does the law of one price hold for the tradable index? Suppose vice versa $\mu^* = \mu$ but $\eta^* \neq \eta$. How does your answer change?

Answer: No, the law of one price does not hold for the tradable price index P_T. The Foreign counterpart of (5) is:

$$P_T^* = \left[\mu^* P_H^{*1-\eta} + (1-\mu^*) P_F^{*1-\eta}\right]^{\frac{1}{1-\eta}}.$$

We know that $P_j = P_j^*$ for $j = \{H, F\}$ so that:

$$P_T^* = \left[\mu^* P_H^{1-\eta} + (1-\mu^*) P_F^{1-\eta}\right]^{\frac{1}{1-\eta}} \neq P_T$$

because $\mu^* \neq \mu$. This result is sufficient for purchasing power parity (PPP) not to hold (i.e. $P \neq P^*$). Similarly, if $\mu^* = \mu$ but $\eta^* \neq \eta$, the law of one price for the tradable consumption basket does not hold, and consequently PPP breaks down.

(f) Define the real exchange rate as $Q \equiv P^*/P$, the terms of trade as $T \equiv P_F/P_H$ and the relative price of non-tradables to tradables as $X \equiv P_N/P_T$. Is $T^* = T$? What about X^*? Derive an expression for the real exchange rate as a function of the terms of trade and the relative price of non-tradables to tradables.

Answer: The law of one price for good H in real terms is:

$$\frac{P_H}{P} = \frac{P_H^*}{P^*}\frac{P^*}{P} = \frac{QP_H^*}{P^*}.$$

We can rewrite the previous expression as:

$$\frac{P_H}{P_T}\frac{P_T}{P} = \frac{QP_H^*}{P_T^*}\frac{P_T^*}{P^*}. \tag{9}$$

We can factor P_H out of (5) to write:

$$\frac{P_T}{P_H} = \left[\mu + (1-\mu)T^{1-\eta}\right]^{\frac{1}{1-\eta}}.$$

Similarly, for the Foreign country, we have:

$$\frac{P_T^*}{P_H^*} = \left[\mu^* + (1-\mu^*)T^{1-\eta}\right]^{\frac{1}{1-\eta}},$$

where we used the fact that $T = P_F/P_H = P_F^*/P_H^*$ because the law of one price holds for individual goods. Factoring out P_T, we can express the relative price of tradables (6) as a function of the relative price of tradables to non-tradables:

$$\frac{P}{P_T} = \left[\gamma + (1-\gamma)\,X^{1-\theta}\right]^{\frac{1}{1-\theta}}.$$

By the same token, we also have:

$$\frac{P^*}{P_T^*} = \left[\gamma + (1-\gamma)\,X^{*1-\theta}\right]^{\frac{1}{1-\theta}}.$$

We can finally solve for the real exchange rate from (9) and substitute for the relative prices derived above as a function of the terms of trade and the relative price of non-tradables

$$Q = \left[\frac{\mu^* + (1-\mu^*)T^{1-\eta}}{\mu + (1-\mu)T^{1-\eta}}\right]^{\frac{1}{1-\eta}}\left[\frac{\gamma + (1-\gamma)\,X^{*1-\theta}}{\gamma + (1-\gamma)\,X^{1-\theta}}\right]^{\frac{1}{1-\theta}}. \tag{10}$$

(g) Take a log-linear approximation of the real exchange rate around a steady state in which relative prices equal one. Compute the response of the real exchange rate to a one percent increase in the terms of trade and in the relative price of non-tradables in each country.

Hint: Up to the a first order approximation, any function $f(x)$ can be written as:

$$f(x) = f(\bar{x}) + f'(\bar{x})(x - \bar{x}),$$

where $\bar{x}$ is the point around which the approximation is taken. Using the formula for the Taylor expansion above, show that up to the first order:

$$\hat{x} \equiv \ln\left(\frac{x}{\bar{x}}\right) = \frac{x - \bar{x}}{\bar{x}}.$$

Answer: To clarify the approximation in the hint, note that we can always rewrite $x = e^{\ln x}$. Let $y = \ln x \Rightarrow x = e^y$. Consider an approximation of x around $\ln \bar{x} = \bar{y}$. Applying the Taylor expansion formula to the first order, we have:

$$x = e^y \approx e^{\bar{y}} + e^{\bar{y}}(y - \bar{y}) = \bar{x} + \bar{x}(\ln x - \ln \bar{x}) \Rightarrow \frac{x - \bar{x}}{\bar{x}} = \ln\left(\frac{x}{\bar{x}}\right).$$

Using the Taylor expansion formula for the real exchange rate, we have:

$$\begin{aligned}
Q - \bar{Q} = & \frac{1}{1-\eta}\left[\frac{\mu^* + (1-\mu^*)\bar{T}^{1-\eta}}{\mu + (1-\mu)\bar{T}^{1-\eta}}\right]^{\frac{1}{1-\eta}-1} \\
& \times \frac{(1-\eta)\bar{T}^{-\eta}\{(1-\mu^*)[\mu + (1-\mu)\bar{T}^{1-\eta}] - (1-\mu)[\mu^* + (1-\mu^*)\bar{T}^{1-\eta}]\}}{[\mu + (1-\mu)\bar{T}^{1-\eta}]^2}(T - \bar{T}) \\
& + \frac{1}{1-\theta}\left[\frac{\gamma + (1-\gamma)\,\bar{X}^{*1-\theta}}{\gamma + (1-\gamma)\,\bar{X}^{1-\theta}}\right]^{\frac{1}{1-\theta}-1} \frac{(1-\gamma)(1-\theta)\bar{X}^{*-\theta}}{\gamma + (1-\gamma)\,\bar{X}^{1-\theta}}(X^* - \bar{X}^*) \\
& - \frac{1}{1-\theta}\left[\frac{\gamma + (1-\gamma)\,\bar{X}^{*1-\theta}}{\gamma + (1-\gamma)\,\bar{X}^{1-\theta}}\right]^{\frac{1}{1-\theta}-1} \frac{[\gamma + (1-\gamma)\,\bar{X}^{*1-\theta}](1-\gamma)(1-\theta)\bar{X}^{-\theta}}{[\gamma + (1-\gamma)\,\bar{X}^{1-\theta}]^2}(X - \bar{X})
\end{aligned}$$

Because we are assuming $\bar{T} = \bar{X} = \bar{X}^* = 1$, we also have $\bar{Q} = 1$. Therefore, we can simplify the previous expression and use the result in the hint as:

$$\hat{Q} = (\mu - \mu^*)\hat{T} + (1-\gamma)(\hat{X}^* - \hat{X}),$$

which corresponds to the expression derived in Lecture 3 when $\mu = \alpha$ and $\mu^* = 1 - \alpha$ (also $\gamma = \omega$). Given our approximation, $\hat{x}$ already measures the percentage deviation (from steady state) of any variable x. Therefore, the effects of a percentage change in the terms of trade and of the relative price of non-tradables are simply given by $\mu - \mu^*$ and $1 - \gamma$, respectively.

2 Dutch Disease and De-Industrialization

Consider a small open economy populated by a large number of identical households with preferences given by:

$$U(c_{T,t}, c_{N,t}) = \sum_{t=0}^{\infty} \beta^t (\ln c_{T,t} + \ln c_{N,t}), \tag{A}$$

where $\beta \in (0,1)$ and $c_{T,t}$ and $c_{N,t}$ represent the consumption of tradable and non-tradable goods, respectively. Households start period zero with initial assets d_{-1}, denominated in units of tradable goods. One unit of asset d_t pays a net interest rate r at maturity. Assume $\beta(1+r) = 1$.

The household is endowed with one unit of time. Tradable and non-tradable goods are produced with technology linear in labour:

$$y_{T,t} = A_T h_{T,t} \qquad \text{and} \qquad y_{N,t} = A_N h_{N,t},$$

where $y_{T,t}$ and $y_{N,t}$ denote output in the two sectors, $h_{T,t}$ and $h_{N,t}$ denote hours worked in the two sectors, and A_T and A_N denote (static) sector-specific productivity parameters. Let p_t denote the relative price of non-tradables to tradables and w_t the wage, both expressed in units of tradable goods.

(a) Write the households' optimisation problem and derive the first order conditions.

Answer: The household maximises (A) subject to:

$$c_{T,t} + p_t c_{N,t} + d_t = (1+r)d_{t-1} + w_t.$$

Let λ_t denote the Lagrange multiplier on the budget constraint. The first order condition for consumption of tradable goods is:

$$\frac{1}{c_{T,t}} = \lambda_t.$$

The first order condition for non-tradable goods is:

$$\frac{1}{c_{N,t}} = \lambda_t p_t.$$

Combining these two expressions, we obtain:

$$c_{T,t} = p_t c_{N,t},$$

that is, the household spends an equal amount of resources on both goods.[1]

The first order condition for assets is:

$$\lambda_t = \beta(1+r)\lambda_{t+1}.$$

[1] We can think of the utility function as a transformation of $U(c_t) = \ln c_t$, where $c_t = c_{T,t} c_{N,t}$.

Since we are assuming that $\beta(1+r)$, we obtain $\lambda_t = \lambda_{t+1} = \lambda$, $\forall t$. In turn, this result implies $c_{T,t} = 1/\lambda = c_T$, $\forall t$ and $p_t c_{N,t} = pc_N$, $\forall t$.

Finally, we can write the intertemporal budget constraint as:

$$c = rd_{-1} + \frac{r}{1+r}\sum_{j=0}^{\infty}\frac{w_{t+j}}{(1+r)^j}.$$

where $c = c_T + pc_N$.

(b) Firms operate in perfect competition. Assuming that in equilibrium both goods are produced in positive but finite quantities, write down the solution for the real wage and the relative price of non-tradables.

Answer: In the tradable sector, profits are:

$$\Pi_{T,t} = y_{T,t} - w_t h_{T,t} = A_T h_{T,t} - w_t h_{T,t}.$$

Because goods are produced in positive finite quantities, $h_{T,t}$ needs to be positive and finite. Therefore, perfect competition implies $w_t = A_T$.

In the non-tradable sector, profits are:

$$\Pi_{Nt} = p_t y_{Nt} - w_t h_{N,t} = p_t A_N h_{N,t} - w_t h_{N,t}.$$

Using the same logic as for the tradable sector, we have:

$$p_t = w_t/A_N = A_T/A_N.$$

(c) Derive the solution for consumption of tradable and non-tradable goods, and for employment in the two sectors.

Answer: Combining the fact that the household spends an equal amount of resources on both goods with the solution for total consumption and the real wage, we can write:

$$c_T = \frac{1}{2}(rd_{t-1} + A_T).$$

Using again the equality of expenditure across sector, we have:

$$c_N = \frac{A_N}{2A_T}(rd_{t-1} + A_T).$$

To derive employment in the non-tradable sector, we use the resource constraint $y_N = c_N$. Substituting the production function and the solution for consumption, we have:

$$A_N h_{N,t} = \frac{A_N}{2A_T}(rd_{t-1} + A_T) \Rightarrow h_N = \frac{1}{2A_T}(rd_{t-1} + A_T).$$

Finally, the resource constraint for the labour market gives:

$$h_T = 1 - \frac{1}{2A_T}(rd_{t-1} + A_T).$$

(d) What is the effect of lower initial assets on consumption of both goods, and employment in both sectors?

Answer: Inspection of the solution reveals that consumption in both sectors will decline, where the factor of proportionality is given by relative productivity:

$$\frac{\partial c_T}{\partial d_{t-1}} = \frac{r}{2} \qquad \text{and} \qquad \frac{\partial c_N}{\partial d_{-1}} = \frac{r}{2}\frac{A_N}{A_T}.$$

A fall (rise) in initial wealth level will results in a fall (rise) in consumption of both goods. This will be split according to the weights in the consumption aggregation (here even), such that expenditure on both goods rises by the same amount.

Employment, however, will behave differently in the two sectors:

$$\frac{\partial h_T}{\partial d_{t-1}} = -\frac{r}{2A_T} < 0 \qquad \text{and} \qquad \frac{\partial h_N}{\partial d_{-1}} = \frac{r}{2A_T} > 0.$$

Therefore, as assets decrease, employment moves more and more to the tradable sector.

This occurs as a fall (rise) in initial wealth leads households to consume less (more) of both products. The only possible way to consume less (more) tradable goods is to produce less (more) of these in equilibrium. Therefore labour supply in this sector falls (increases).

There are two possible ways to alter the availability of tradable consumption goods: via production or through imports. With a lower level of initial resources the country imports less. Therefore production in the traded goods sector will increase to offset this effect.

(e) We say that an economy "de-industrialises" (or suffers from the "Dutch disease") when the share of employment in the tradable sector falls permanently. Show that there exists a level of net foreign assets $\bar{d}_{-1}$ above which this economy becomes completely de-industrialised.

Answer: Employment in the tradable sector is positive if:

$$1 - \frac{1}{2A_T}(rd_{t-1} + A_T) > 0.$$

Solving for net foreign assets, we obtain that this economy de-industrializes completely when:

$$\bar{d}_{t-1} \geq \frac{A_T}{r}.$$

(f) Derive an expression for the real exchange rate (which we will define as the relative price of non-traded to traded goods, p_t) and the real wage when the economy is completely de-industrialised. Compare the solution to the previous case.

Answer: In this case, we cannot use the zero profit condition in the tradable sector to pin down the real wage. While the first order conditions for consumption and savings are the same, we now have $h_T = 0$ and $h_N = 1$. The zero-profit condition in the non-tradable sector is:

$$p_t A_N = w_t.$$

We can seek a solution with constant relative price, so that now consumption in the two sectors is:

$$c_T = \frac{1}{2}(rd_{t-1} + pA_N),$$

and

$$c_N = \frac{1}{2p}(rd_{t-1} + pA_N).$$

The resource constraint in the non-tradable sector now pins down the relative price of non-tradables:

$$A_N = \frac{1}{2p}(rd_{t-1} + pA_N) \Rightarrow p = \frac{rd_{-1}}{A_N} \Rightarrow w = rd_{t-1}.$$

Looking at the real exchange rate, the productivity of the tradable sector is now irrelevant. This should not be surprising, as the domestic economy no longer produces tradable goods. What is relevant, instead, is the ratio between how many assets the economy has and how productive the non-tradable sector is. Similarly, the real wage only depends on the economy's wealth.