International Economics, Lecture 4
International Business Cycles

Dan Wales
University of Cambridge

London School of Economics
22 October, 2018

Plan

- How to Measure Business Cycles
- International Business Cycle Facts
- Can Basic Economic Theory Explain These Movements?
- Quantity Puzzle
- Relative Price Puzzle
- Summary

Business Cycles - Two (related) Definitions

- Statistical, due to Burns and Mitchell (1946): An empirical characterisation and definition.

  "Business cycles are a type of fluctuation found in the aggregate economic activity of nations that organize their work mainly in business enterprises: a cycle consists of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions, and revivals which merge into the expansion phase of the next cycle; this sequence of changes is recurrent but not periodic; in duration business cycles vary from more than one year to ten or twelve years; they are not divisible into shorter cycles of similar character with amplitudes approximating their own.' (pg 3).

- Theoretical, due to Lucas (1975, 1977): Perturbations of output around its trend due to fundamental 'shocks'.

Summarising Business Cycles

- Business cycles are usually summarised with a set of moment conditions, including:
  - Volatilities of time series data (standard deviations).
  - Comovements (correlations, serial correlations).
- When analysing international business cycles we may add:
  - **Relative** volatilities. Do business cycles look similar across countries?
  - **Relative** comovements. Are macroeconomic variables correlated between countries?
  - Usually compared to **the US economic cycle**.

How to Measure Business Cycles

- Key idea is to separate economic time series data, y_t, into a **trend** component, y_t^T, and a **cyclical** component, y_t^C:

$$y_t = y_t^T + y_t^C,$$

- Business cycle variations are then defined as movements in the variable of choice **after accounting for its trend**.

$$y_t^C = y_t - y_t^T,$$

- Two issues:
  - Which data to use?
  - How to separate trend from cycle?

Which Data to Use?

- Annual.
  - \+ Greater availability.
  - \- May obscure the business cycle.
- Quarterly.
  - \- Across countries, very limited availability (particularly for developing countries).
  - \+ Will capture shorter duration business cycles.
- Does it really matter?
  - We often seek to increase the number of available data points, $N \times T$. We have panel data here. Annual data may increase the cross sectional dimension (N), while quarterly data may increase the time dimension (T).

Separating Trend from Cycle - Numerous Options

- Hodrick-Prescott filtering.
- Log-quadratic detrending.
- Other options include first differencing, band-pass filters, etc...

HP Filter - Definition

- The most popular decomposition.
- Somewhat agnostic way of removing trend components from time series data. Suppress low frequency movements to highlight the business cycle.
- y_t^T is the solution which minimises the loss function:

$$
\begin{aligned}
L &= \sum_{t=1}^{T}(y_t - y_t^T)^2 + \lambda \sum_{t=2}^{T-1}[(y_{t+1}^T - y_t^T) - (y_t^T - y_{t-1}^T)]^2, \\
&= \sum_{t=1}^{T}(y_t^C)^2 + \lambda \sum_{t=2}^{T-1}[\Delta y_{t+1}^T - \Delta y_t^T]^2,
\end{aligned}
$$

such that you penalise both cyclical deviations and movements in trend growth rate.

HP Filter - Implementation

- The appropriateness of the HP filter and how to choose the smoothness parameter, λ, are both complex theoretical questions... which we will sidestep entirely.
  - ... but beware. Disagreement exists, see Hamilton (2017).
- Most researchers use $\lambda = 1600$ for quarterly data and $\lambda = 100$ for annual.
- Implementation on computer software is mechanical. MatLab, Stata, EViews etc.

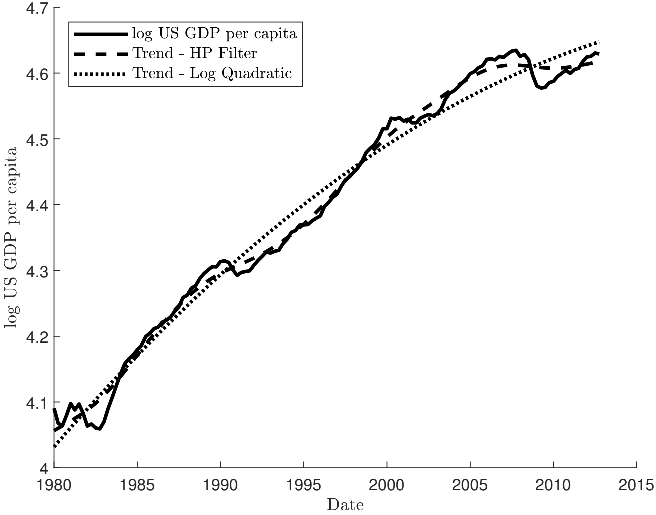
Log-Quadratic Detrending - Definition

- Start with variables in logs.
- Then run the regression:

$$y_t = \underbrace{\alpha + \beta t + \gamma t^2}_{y_t^T} + \underbrace{\varepsilon_t}_{y_t^C},$$

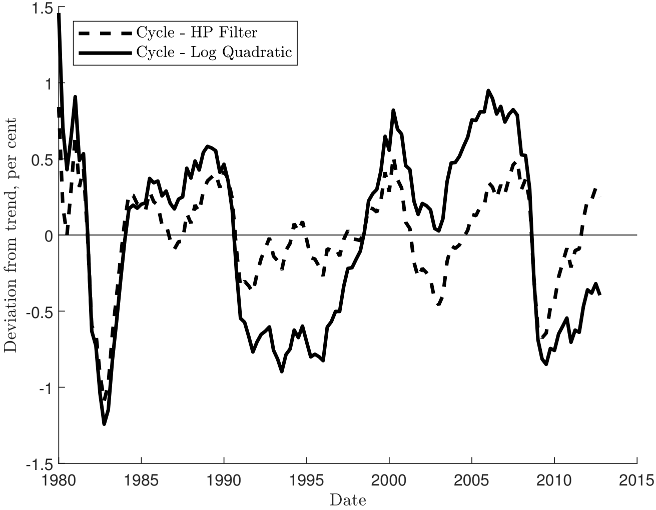
  to remove the time trend and obtain estimates for $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\gamma}$.
- Finally, set $y_t^T = \hat{\alpha} + \hat{\beta} t + \hat{\gamma} t^2$ and $y_t^C = \hat{\varepsilon}_t$.

Log US GDP Per Capita - Trend



Source: Schmitt-Grohé and Uribe (2017) and own calculations.

Log US GDP Per Capita - Cycle



Source: Schmitt-Grohé and Uribe (2017) and own calculations.

International Business Cycle Facts I

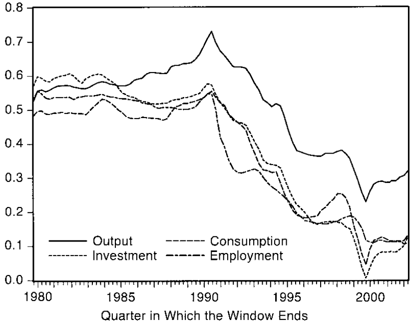
1. Across countries, consumption (non-durable and services) is less volatile than output.
    - Motivation for consumption smoothing.
    - Note: SGU include durable consumption (SGU fact 2).
2. Consumption is less correlated than output.
    - High correlation of macroeconomic variables between countries.
    - Quantity puzzle.

	CAN	FRA	GER	ITA	JAP	UK	US
σ_y	1.60	0.96	1.46	1.80	1.32	1.56	2.00
σ_c/σ_y	0.86	0.95	0.77	0.78	1.10	1.14	0.71
ρ_{y^{US},y^X}	0.81	0.46	0.85	0.49	0.66	0.64	1.00
ρ_{c^{US},c^X}	0.46	0.42	0.64	0.04	0.49	0.42	1.00

Source: Baxter (1995), Table 2.1.

Are These Cross-Country Relationships Stable?

- No, they depend critically on the **precise shocks** which arise.
- Interaction with asset market structure (ability to smooth consumption).



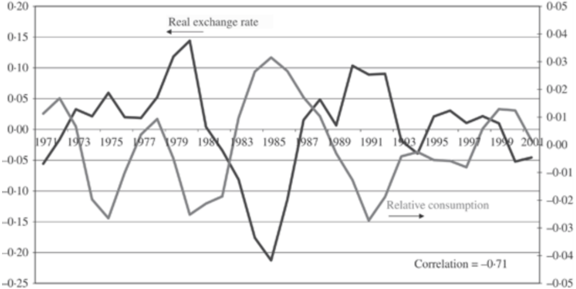
Source: Heathcote and Perri (2003).

International Business Cycle Facts II

3. Across countries, the correlation between relative consumption and the real exchange rate is low, and even negative.
    - "Backus and Smith (1993) problem". Consider:

$$\frac{u_c(C_t^*)}{u_c(C_t)} = Q_t \rightarrow \sigma \ln \frac{C_t}{C_t^*} = \ln Q_t,$$

    - Efficient: household with cheaper basket consumes more.



The real exchange rate and relative consumption are constructed using trade weights as described in the data appendix. Both series are logged and HP-filtered.

Source: Corsetti et al. (2008).

International Business Cycle Facts III

4. Although both imports and exports are procyclical, net exports are countercyclical (imports are more procyclical, than exports).
    - Note: SGU add current account trade-balance-to-output ratio and current-account-to-output ratio (SGU fact 5).

	CAN	FRA	GER	ITA	JAP	UK	US
ρ_{y^X, nx^X}	-0.35	-0.33	-0.21	-0.73	-0.32	-0.40	-0.37

Source: Baxter (1995), Table 2.1.

International Business Cycle Facts IV

5. Excess volatility of poor and emerging countries.
    - Business cycles in rich countries are around half as volatile as in emerging or poor countries (SGU fact 8).
6. Less consumption smoothing in poor and emerging countries.
    - The relative consumption volatility is higher in poor and emerging countries than the rich (SGU fact 9).

	All	Poor	Emerging	Rich
σ_y	3.79	4.12	3.98	2.07
σ_c/σ_y	1.08	1.09	1.23	0.87

Source: Schmitt-Grohé and Uribe (2017), Table 1.3.

Can Basic Economic Theory Explain These Movements?

- We will attempt to explain the international business cycle facts using two models, and will encounter two puzzles.
- **Quantity puzzle**. In the data consumption is less correlated that output, but not in the models.
  - If asset markets pool consumption risk, then consumption should move similarly across countries.
  - Lower output in one country leads to a **smaller** change in consumption as they borrow from abroad.
- **Relative price puzzle**. In the data the changes in terms of trade are persistent, and highly volatile. But the models will be unable to replicate these volatilities.

Model 1 - Backus, Kehoe, and Kydland (1992)

- Two country version of Kydland and Prescott (1982) closed-economy with "time to build".
- Output is **endogenous**.
- Complete asset markets.
- One consumption good.
- Largely the same across countries, with two important differences:
    - Labour is cannot move between countries.
    - Production receives a country-specific technology shock.
- Will highlight the **quantity puzzle**.

Households

- Identical preferences across countries $i = \{H, F\}$:

$$U_{i,t} = \mathbb{E}_t \left[\sum_{s=t}^{\infty} \frac{(c_{i,s}^{\mu} \ell_{i,s}^{1-\mu})^{\gamma}}{\gamma} \right],$$

where $c_{i,t}$ represents consumption, $\ell_{i,t} = 1 - n_{i,t}$ represents leisure and parameters are restricted with $0 < \mu < 1$ and $\gamma < 1$.

  - Ignore the non-time separability of the utility function in the original paper.
  - Notice in the limit $\gamma \to 1$, these preferences become:

$$U_{i,t} = \mathbb{E}_t \left[\sum_{s=t}^{\infty} \mu \ln(c_{i,s}) + (1-\mu) \ln(\ell_{i,s}) \right].$$

Firms

- The production displays constant returns to scale:

  $$y_{i,t} = z_{i,t} F(k_{i,t}, n_{i,t}) = z_{i,t} k_{i,t}^{\theta} n_{i,t}^{1-\theta},$$

  where θ is the Cobb-Douglas production parameter $k_{i,t}$ is the level of capital input and $z_{i,t}$ is an exogenous technology process.

- Capital follows an accumulation process:

  $$k_{i,t+1} = (1-\delta) k_{i,t} + s_{i,t}^{1},$$

  where $s_{i,t}^{1}$ represent inventories with:

  $$s_{i,t+1}^{j} = s_{i,t}^{j+1},$$

Time to Build

- Multiple periods required to build new capital. Only finished capital forms a part of the productive capital stock.
- This structure incorporates time to build over J periods. Investment is given as:

$$x_{i,t} = \sum_{j=1}^{J} \phi_j s_{i,t}^{j},$$

with $\phi_j = \frac{1}{J}$, such that investment today makes an even contribution to the level of stocks for the next J periods.

Constraints

- One final equation, the (global) resource constraint:

$$\sum_i y_{i,t} = \sum_i c_{i,t} + x_{i,t} + g_{i,t},$$

- Four exogenous shock processes:

$$\begin{aligned} z_t &= A z_{t-1} + \varepsilon_t^z, \\ g_t &= B g_{t-1} + \varepsilon_t^g, \end{aligned}$$

where $\varepsilon_t^k \sim N(0, V_k)$ and $z_t, g_t, \varepsilon_t^z, \varepsilon_t^g$ are vectors while A, B, V_k are matrices.

Calibration

- Target relationship between US and RoW.

Parameters

Preferences	
Household discount factor	$\beta = 0.99$
Consumption utility-weight	$\mu = 0.34$
1-Coeff. of RRA	$\gamma = -1$
Time-separability (off)	$\alpha = 1$
Technology	
Capital income share	$\theta = 0.36$
Inventory param 1	$\nu = 3$
Inventory param 2	$\sigma = 0.01$
Depreciation rate	$\delta = 0.025$
Time to build	$J = 4$
Shocks	
$A = \begin{bmatrix} 0.906 & 0.088 \\ 0.088 & 0.906 \end{bmatrix}$	$V_z = \sigma_z^2 \begin{bmatrix} 1 & 0.258 \\ 0.258 & 1 \end{bmatrix}$
$\sigma_z^2 = 0.00852^2$	
Government spending	$g_{i,t} = 0$

Source: Backus et al. (1992), Table 3.

Aside: The Second Welfare Theorem of Economics*

- Provided we have:
    - Convex production set.
    - Convex preferences.
    - No boundary issues.
- We can then any competitive equilibrium may be computed as the solution to the (associated) social planner's problem.

Solution

- We have no distortions and complete asset markets, thus equivalence of competitive equilibrium and **Pareto optima** (second welfare theorem holds).
- "Easier" way to calculate the solution - just solve the Social Planner's problem:

  $$\max \phi U_{H,t} + (1-\phi) U_{F,t},$$

  subject to the production and budget conditions, above. Set $\phi = \frac{1}{2}$.
- Take log-linear approximation around the steady state.
- Investigate the impact of productivity shocks.

Explicit Formulation of the Model I

- **Ignore time to build**, the model then consists of 11 variables:

$$\{c_{i,t}, n_{i,t}, k_{i,t+1}, x_{i,t}, z_{i,t}\} \quad \text{and} \quad \{\lambda_t\}.$$

- We therefore require 11 equations to solve the model.
- The social planner chooses:

$$\{c_{i,t}, n_{i,t}, k_{i,t+1}\}.$$

  to maximise welfare, giving us 6 first order conditions.
- Additional constraints add the remaining 5:
  - Global production constraint.
  - Capital accumulation equation.
  - The exogenous AR(1) technology process.

Explicit Formulation of the Model II

- Formally, the Lagrangian of the problem may be written as:

$$\begin{aligned}\mathcal{L} = \max \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \Big[& u_{H,s}(c_{H,s},(1-n_{H,s})) + u_{F,s}(c_{F,s},(1-n_{F,s})) \\ & + \lambda_s \Big(z_{H,s} F(k_{H,s}, n_{H,s}) + z_{F,s} F(k_{F,s}, n_{F,s}) \\ & + (1-\delta) k_{H,s} + (1-\delta) k_{F,s} \\ & - c_{H,s} - c_{F,s} - k_{H,s+1} - k_{F,s+1} \Big) \Big],\end{aligned}$$

where functional forms are given above, with:

$$\begin{aligned} F(k_{i,t}, \ell_t) &= k_{i,t}^{\theta} n_{i,t}^{1-\theta}, \\ u_{i,t} &= \frac{(c_{i,t}^{\mu}(1-n_{i,t})^{1-\mu})^{\gamma}}{\gamma}. \end{aligned}$$

Equilibrium Conditions I

- First 6 FOCs, with respect to (respectively):

$$\{c_{H,t}, c_{F,t}, n_{H,t}, n_{F,t}, k_{H,t+1}, k_{F,t+1}\}.$$

$$\lambda_t = \mu c_{H,t}^{\mu\gamma-1}(1-n_{H,t})^{\gamma(1-\mu)},$$
$$\lambda_t = \mu c_{F,t}^{\mu\gamma-1}(1-n_{F,t})^{\gamma(1-\mu)},$$

$$\lambda_t z_{H,t}(1-\theta)k_{H,t}^{\theta}n_{H,t}^{-\theta} = (1-\mu)c_{H,t}^{\mu\gamma}(1-n_{H,t})^{\gamma(1-\mu)-1},$$
$$\lambda_t z_{F,t}(1-\theta)k_{F,t}^{\theta}n_{F,t}^{-\theta} = (1-\mu)c_{F,t}^{\mu\gamma}(1-n_{F,t})^{\gamma(1-\mu)-1},$$

$$\lambda_t = \beta\,\mathbb{E}_t[(1-\delta)\lambda_{H,t+1} + \lambda_{H,t+1}\theta z_{H,t+1}k_{H,t+1}^{\theta-1}n_{H,t+1}^{1-\theta}],$$
$$\lambda_t = \beta\,\mathbb{E}_t[(1-\delta)\lambda_{F,t+1} + \lambda_{F,t+1}\theta z_{F,t+1}k_{F,t+1}^{\theta-1}n_{F,t+1}^{1-\theta}].$$

Equilibrium Conditions II

- Remaining conditions represent constraints, starting with (global) production:

$$z_{H,t}k_{H,t}^{\theta}n_{H,t}^{1-\theta}+z_{F,t}k_{F,t}^{\theta}n_{F,t}^{1-\theta}=c_{H,t}+c_{F,t}+x_{H,t}+x_{F,t},$$

$$x_{H,t}=k_{H,t+1}-(1-\delta)k_{H,t}, \qquad \text{(k acc., H)}$$

$$x_{F,t}=k_{F,t+1}-(1-\delta)k_{F,t}, \qquad \text{(k acc., F)}$$

$$z_{H,t}=a_{1,1}z_{H,t-1}+a_{1,2}z_{F,t-1}+\varepsilon_{H,t}, \qquad \text{(TFP, H)}$$

$$z_{F,t}=a_{2,2}z_{F,t-1}+a_{2,1}z_{H,t-1}+\varepsilon_{H,t}. \qquad \text{(TFP, F)}$$

- "Time to build" alters the capital accumulation equation, while also introducing an additional endogenous variable (stocks) and investment constraint.

Net Exports

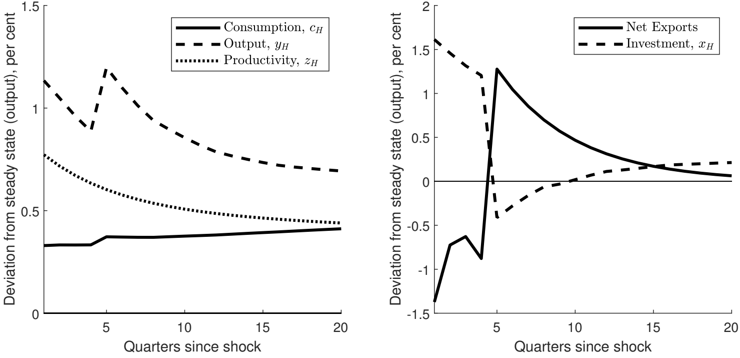
- In a second stage, net exports may then be computed as:

$$nx_{i,t} = y_{i,t} - c_{i,t} - x_{i,t}.$$

where $y_{i,t} = z_{i,t} k_{i,t}^{\theta} n_{i,t}^{1-\theta}$.

Response to a Technology Shock - Home

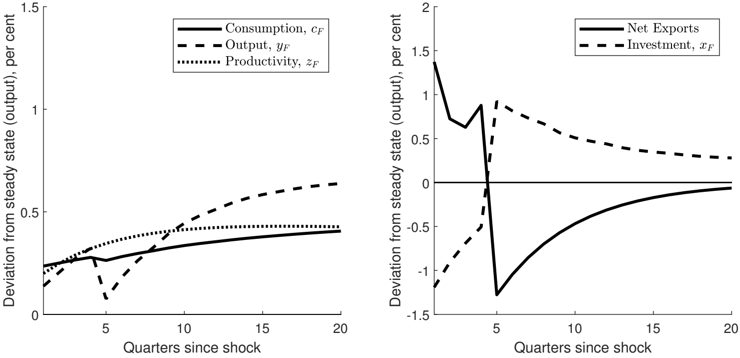
- $z_{i,t}$ increases, then gradual fall to the new long run level.
- Investment, output and consumption all increase.
- Increase in investment and consumption $>$ output, as easy to fund in open economy (can borrow from abroad).
- Net exports falls.



Source: Replication of Backus et al. (1992), Figure 2.

Response to a Technology Shock - Foreign

- $z_{i,t}$ increase eventually spills over to $z^*_{i,t}$.
- Initially net exports increase, as all resources transferred to the Home country. “Make hay while the sun shines”.
- Consumption increases immediately, as both countries share in the higher productivity (risk sharing).



Source: Replication of Backus et al. (1992), Figure 2.

What Went In? What Came Out?

- Consumption smoothing. Gives procyclical NX.
- Investment - a chance for countercyclical NX.
- Complete asset markets. Wealth and substitution effects are shared across countries. Hard wired into the model the correlation of consumption across countries.

Simulations

- Method:
  - Simulate 50 samples of 100 observations ($\approx$ actual sample length).
  - HP filter simulated data.
- Report average statistics, compared to the data. Consider:
  - Volatilities.
  - Relative volatilities.
  - Contemporaneous correlations with output.
  - Cross-country correlations.

Simulation Results - Volatilities, (σ_X)

- Output close to perfect.
- Consumption "too smooth".
- Investment and NX "too volatile".

	σ_y	σ_c	σ_I
US Data	1.71	0.84	5.38
BKK	1.55	0.62	16.91
Dynare	1.49	0.61	36.3

Source: Backus et al. (1992) Tables 1 and 4, and Dynare replication.

Simulation Results - Relative Volatilities, (σ_x/σ_y)

- Consumption close to perfect.
- Investment far too volatile.

	σ_c/σ_y	σ_I/σ_y
US Data	0.49	3.15
BKK	0.40	10.94
Dynare	0.41	24.6

Source: Backus et al. (1992) Tables 1 and 4, and Dynare replication.

Simulation Results - Cross-country correlations

- Output negatively correlated! (Significantly positively correlated in the data).
- Consumption too highly correlated (lower than output correlation in the data). **Quantity puzzle**.

	ρ_{y^H,c^H}	ρ_{y^H,I^H}	ρ_{y^H,y^F}	ρ_{c^H,c^F}
US Data	0.76	0.90	0.33*	0.21*
BKK	0.79	0.27	-0.18	0.88
Dynare	0.83	0.26	-0.12	0.89

Source: Backus et al. (1992) Tables 1 and 4, and Dynare replication.
*Simple average from Backus et al. (1992), Table 2.

One Solution: Transport Costs

- Try the model again, adding ad hoc trading costs.
- World budget constraint becomes:

$$\sum_i y_{i,t} = \sum_i c_{i,t} + x_{i,t} + g_{i,t} + \tau nx_{i,t}^2,$$

- Increasing investment by importing goods becomes expensive, calibrate to become 1% of output value ($\tau = 0.1y$).
- Lower response of net exports, and hence investment.
- Does not solve the consumption quantity puzzle (worsens).
- Alternative solution: Incomplete asset markets (Lecture 6).

A Note on the Classical RBC Literature

- Three main contributions introduced by Kydland and Prescott (1982) (and followed in Backus et al. (1992)):
  1. Utility function is not time-separable, with current leisure decisions depending on those made in the past.
     - Higher intertemporal elasticity of substitution for leisure increases the volatility of employment.
  2. "Wine is not made in a day" / "time to build".
     - Increases the persistence of output responses.
  3. Inventories.
- None followed in much of the subsequent literature.
  - 1, valuable and important but not theoretically motivated here.
  - 2 + 3 add relatively little volatility or persistence, as investment is small relative to the level of capital stock.

Model 2 - Backus, Kehoe, and Kydland (1994)

- So far we had focused on quantities in a single good economy.
- Key difference: each country now produces an imperfectly substitutable good.
    - Intermediate inputs for C, I and G.
- Otherwise, many of the same features.
- Complete asset markets assumption is maintained.
- Will highlight the **relative price puzzle puzzle**.

Data: TOT

- TOT is significantly more volatile than output, $(\sigma_{y_t^x, \mathcal{T}_t^x})$.
- Highly persistent variable.
- Mixed evidence on correlation between TOT and output $(\rho_{y_t^x, \mathcal{T}_t^x})$ where the sign and magnitude vary considerably. (Acyclical?)
- Also for correlation between TOT and nx $(\rho_{nx_t^x/y_t^x, \mathcal{T}_t^x})$ where the US may be an exception.

	CAN	FRA	GER	ITA	JAP	UK	US
$\sigma_{\mathcal{T}}/\sigma_y$	1.61	3.89	1.76	2.08	3.49	1.81	1.60
$\rho_{y_t^x, \mathcal{T}_t^x}$	-0.10	-0.12	-0.13	0.38	-0.12	0.19	0.03
$\rho_{nx_t^x/y_t^x, \mathcal{T}_t^x}$	0.04	-0.50	0.00	-0.66	-0.47	-0.54	0.27

Source: Backus et al. (1994), Table 1.

The J-curve I

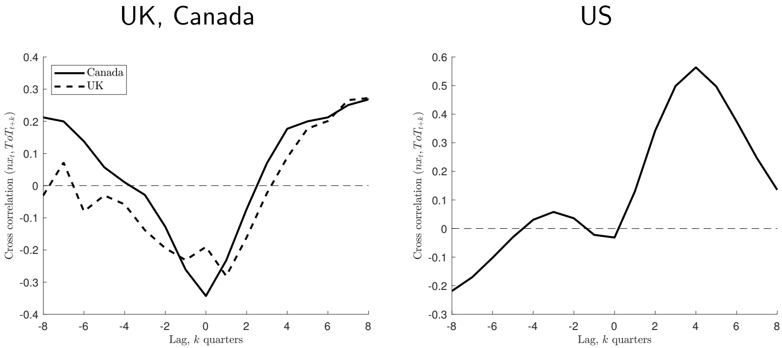
- However, a clearer pattern emerges for $\rho(\mathcal{T}_t, nx_{t+k}/y_t)$.
- Given the net exports definition (in price of home good):

$$nx_t = X_t - \mathcal{T}_t M_t.$$

- Suppose there is a terms of trade deterioration ($\mathcal{T}_t \uparrow$).
- In the short run, households' consumption reacts little, with constant imports and exports, $\bar{X}$ and $\bar{M}$ (low elasticities < 1). Net exports therefore fall, $nx_t \downarrow$.
- After some time, households' react to the new relaive prices. Consumption rebalances between goods (high elasticity > 1). Net exports retrench their initial fall, $nx_t \uparrow$.
- Unfavourable TOT movements, $\mathcal{T}_t \uparrow$, are associated with initially lower nx, which reverses after around 2 years.
- Result of nominal stickiness? Delivery lags?

The J-curve II

- Empirically, this patterns appears strongly for most economies.
- Though still less apparent in the US (post 1972) data.



UK, Canada

US

Sources: BEA, CANSIM and ONS.

Changes to the Model I

- Clearly, we are going to need 2 goods.
- Use the familiar Armington (1969) aggregation procedure for $c_{i,t}$, $x_{i,t}$ and $g_{i,t}$:

  $$\Psi(a_t, b_t) = \left[\omega^{\frac{1}{\sigma}} a_t^{\frac{\sigma-1}{\sigma}} + (1-\omega)^{\frac{1}{\sigma}} b_t^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},$$

  where a_t is produced in Home and b_t in foreign (Lecture 3).
- Terms of Trade:

  $$\mathcal{T}_t \equiv \frac{P_{F,t}}{P_{H,t}}.$$

Changes to the Model II

- Account for this change in BCs and NX definitions.
- Two goods market clearing conditions (for $i = \{H, F\}$):

$$y_{i,t} = c_{i,t} + c^*_{i,t} + x_{i,t} + x^*_{i,t} + g_{i,t} + g^*_{i,t}.$$

- Global budget constraint becomes (using $\mathcal{T}_t$):

$$\begin{aligned} y_{H,t} + \mathcal{T}_t y_{F,t} = {} & c_{H,t} + c^*_{H,t} + x_{H,t} + x^*_{H,t} + g_{H,t} + g^*_{H,t} \\ & + \mathcal{T}_t(c_{F,t} + c^*_{F,t} + x_{F,t} + x^*_{F,t} + g_{F,t} + g^*_{F,t}). \end{aligned}$$

- Net trade also accounts for relative prices:

$$nx_t = c^*_{H,t} + x^*_{H,t} + g^*_{H,t} - \mathcal{T}_t(c_{F,t} + x_{F,t} + g_{F,t}).$$

Calibration

- As previous, but add:

Parameters

Trade elasticity	$\sigma = 1.5$
1-Import share	$\omega = 0.85$

Source: Backus et al. (1994), Table 2.

- So not quite Cobb-Douglas here.
- Turn off inventories and “time to build”.
- Also remove government spending, for now.

Explicit Formulation of the Model

- We again look for the social planners solution to the problem:

$$\begin{aligned}\mathcal{L} = \max \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \Big[& U_{H,s}(c_{H,s}, (1-n_{H,s})) + U_{F,s}(c_{F,s}, (1-n_{F,s})) \\ & + \lambda_{H,s}[\Psi(a_{H,s}, b_{H,s}) + (1-\delta)k_{H,s} - c_{H,s} - k_{H,s+1}] \\ & + \lambda_{F,s}[\Psi(a_{F,s}, b_{F,s}) + (1-\delta)k_{F,s} - c_{F,s} - k_{F,s+1}] \\ & + \Lambda_{H,s}[z_{H,s}F(k_{H,s}, n_{H,s}) - a_{H,s} - a_{F,s}] \\ & + \Lambda_{F,s}[z_{F,s}F(k_{F,s}, n_{F,s}) - b_{F,s} - b_{F,s}] \Big],\end{aligned}$$

where the functional forms given above, where:

$$\begin{aligned}\Psi(a_{i,t}, b_{i,t}) &= \left[\omega^{\frac{1}{\sigma}} a_{i,t}^{\frac{\sigma-1}{\sigma}} + (1-\omega)^{\frac{1}{\sigma}} b_{i,t}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \\ F(k_{i,t}, n_t) &= k_{i,t}^{\theta} n_{i,t}^{1-\theta}, \\ U_{i,t} &= \mathbb{E}_t \left[\sum_{s=t}^{\infty} \frac{(c_{i,s}^{\mu}(1-n_{i,s})^{1-\mu})^{\gamma}}{\gamma}\right].\end{aligned}$$

Solution

- The model then consists of 18 variables:

$$\{c_{i,t}, n_{i,t}, k_{i,t+1}, a_{i,t}, b_{i,t}, x_{i,t}, \lambda_{i,t}, \Lambda_{i,t}, z_{i,t}\}.$$

- We therefore require 18 equations to solve the model.
- The social planner chooses:

$$\{c_{i,t}, n_{i,t}, k_{i,t+1}, a_{i,t}, b_{i,t}\}.$$

  to maximise welfare, giving us 10 first order conditions.
- Additional constraints add the remainder:
  - Production, consumption and capital accumulation.
  - The exogenous AR(1) technology process.

Equilibrium Conditions I

- First 6 FOCs, with respect to (respectively):

$$\{c_{H,t}, c_{F,t}, n_{H,t}, n_{F,t}, k_{H,t+1}, k_{F,t+1}\}.$$

$$\lambda_{H,t} = \mu c_{H,t}^{\mu\gamma-1}(1 - n_{H,t})^{\gamma(1-\mu)},$$
$$\lambda_{F,t} = \mu c_{F,t}^{\mu\gamma-1}(1 - n_{F,t})^{\gamma(1-\mu)},$$

$$\Lambda_{H,t} z_{H,t}(1-\theta) k_{H,t}^{\theta} n_{H,t}^{-\theta} = (1-\mu) c_{H,t}^{\mu\gamma}(1 - n_{H,t})^{\gamma(1-\mu)-1},$$
$$\Lambda_{F,t} z_{F,t}(1-\theta) k_{F,t}^{\theta} n_{F,t}^{-\theta} = (1-\mu) c_{F,t}^{\mu\gamma}(1 - n_{F,t})^{\gamma(1-\mu)-1},$$

$$\lambda_{H,t} = \beta\, \mathbb{E}_t[(1-\delta)\lambda_{H,t+1} + \Lambda_{H,t+1}\theta z_{H,t+1} k_{H,t+1}^{\theta-1} n_{H,t+1}^{1-\theta}],$$
$$\lambda_{F,t} = \beta\, \mathbb{E}_t[(1-\delta)\lambda_{F,t+1} + \Lambda_{F,t+1}\theta z_{F,t+1} k_{F,t+1}^{\theta-1} n_{F,t+1}^{1-\theta}].$$

Equilibrium Conditions II

- Remaining 4 FOCs, with respect to (respectively):

$$\{a_{H,t}, a_{F,t}, b_{H,t}, b_{F,t}\}.$$

$$\Lambda_{H,t} = \lambda_{H,t}\omega^{\frac{1}{\sigma}} a_{H,t}^{-\frac{1}{\sigma}} \left[\omega^{\frac{1}{\sigma}} a_{H,t}^{\frac{\sigma-1}{\sigma}} + (1-\omega)^{\frac{1}{\sigma}} b_{H,t}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}},$$

$$\Lambda_{H,t} = \lambda_{F,t}(1-\omega)^{\frac{1}{\sigma}} a_{F,t}^{-\frac{1}{\sigma}} \left[(1-\omega)^{\frac{1}{\sigma}} a_{F,t}^{\frac{\sigma-1}{\sigma}} + \omega^{\frac{1}{\sigma}} b_{F,t}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}},$$

$$\Lambda_{F,t} = \lambda_{H,t}(1-\omega)^{\frac{1}{\sigma}} b_{H,t}^{-\frac{1}{\sigma}} \left[\omega^{\frac{1}{\sigma}} a_{H,t}^{\frac{\sigma-1}{\sigma}} + (1-\omega)^{\frac{1}{\sigma}} b_{H,t}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}},$$

$$\Lambda_{F,t} = \lambda_{F,t}\omega^{\frac{1}{\sigma}} b_{F,t}^{-\frac{1}{\sigma}} \left[(1-\omega)^{\frac{1}{\sigma}} a_{F,t}^{\frac{\sigma-1}{\sigma}} + \omega^{\frac{1}{\sigma}} b_{F,t}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}}.$$

Equilibrium Conditions III

- 8 constraints, starting with expenditure:

$$
\begin{aligned}
c_{H,t} + x_{H,t} &= \left[\omega^{\frac{1}{\sigma}} a_{H,t}^{\frac{\sigma-1}{\sigma}} + (1-\omega)^{\frac{1}{\sigma}} b_{H,t}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \\
c_{F,t} + x_{F,t} &= \left[(1-\omega)^{\frac{1}{\sigma}} a_{F,t}^{\frac{\sigma-1}{\sigma}} + \omega^{\frac{1}{\sigma}} b_{F,t}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},
\end{aligned}
$$

$$
\begin{aligned}
z_{H,t} k_{H,t}^{\theta} n_{H,t}^{1-\theta} &= a_{H,t} + a_{F,t}, && \text{(Production, H)} \\
z_{F,t} k_{F,t}^{\theta} n_{F,t}^{1-\theta} &= b_{H,t} + b_{F,t}, && \text{(Production, F)}
\end{aligned}
$$

$$
\begin{aligned}
x_{H,t} &= k_{H,t+1} - (1-\delta) k_{H,t}, && \text{(k acc., H)} \\
x_{F,t} &= k_{F,t+1} - (1-\delta) k_{F,t}, && \text{(k acc., F)}
\end{aligned}
$$

$$
\begin{aligned}
z_{H,t} &= a_{1,1} z_{H,t-1} + a_{2,1} z_{F,t-1} + \varepsilon_{H,t}, && \text{(TFP, H)} \\
z_{F,t} &= a_{2,2} z_{F,t-1} + a_{2,1} z_{H,t-1} + \varepsilon_{H,t}. && \text{(TFP, F)}
\end{aligned}
$$

Simulations

- Same method as before:
  - Simulate 50 samples of 100 observations ($\approx$ actual sample length).
  - HP filter simulated data.
- Report average statistics, compared to the data. Consider:
  - Relative volatilities.
  - Contemporaneous correlations with output.
  - Cross-country correlations.
- New part → relative prices (volatility and correlation).

Simulation Results - Relative Volatilities, (σ_x/σ_y)

- Improves upon earlier results by lowering relative volatility of investment.

	σ_c/σ_y	σ_I/σ_y
US Data	0.49	3.15
BKK 92		
Original	0.40	10.94
Dynare	0.41	24.6
BKK 94		
Original	0.47	3.48
Dynare	0.43	2.91

Source: Backus et al. (1992) Tables 1 and 4, Backus et al. (1994) Table 4 and Dynare replications.

Simulation Results - Correlations

- Helps (a small amount) with the **quantity puzzle**.
- Increases output correlation, lowers consumption correlation.

	ρ_{y^H,c^H}	ρ_{y^H,I^H}	ρ_{y^H,y^F}	ρ_{c^H,c^F}
US Data	0.76	0.90	0.33*	0.21*
BKK 92				
Original	0.79	0.27	-0.18	0.88
Dynare	0.83	0.26	-0.12	0.89
BKK 94				
Original	0.88	0.93	0.02	0.77
Dynare	0.84	0.97	-0.07	0.82

Source: Backus et al. (1992) Tables 1 and 4, Backus et al. (1994) Table 4 and Dynare replication. *Simple average from Backus et al. (1992), Table 2.

Simulation Results - Relative Price Puzzle

- **Relative price puzzle**. Volatility - far too low. Even US data is around 4.5 times larger than model generates.
- Autocorrelation (persistence) is similar to the data, inherited from the technology shock.

	$\sigma_{\mathcal{T}}/\sigma_{y}$	$\rho_{\mathcal{T}_t,\mathcal{T}_{t-1}}$	$\rho_{y,\mathcal{T}}$	$\rho_{nx,\mathcal{T}}$
US Data	1.60	0.80	0.03	0.27
BKK 94	0.35	0.88	0.49	-0.41
Dynare	0.28	0.83	0.46	-0.32

Source: Backus et al. (1994) Tables 1 and 3, and Dynare replication.

Simulation Results - Net Exports

- Picture for net exports is very good.
- Countercyclical.
- Close to observed volatility.
- Close to observed (high) persistence.

	σ_{nx}/σ_y	$\rho_{nx_t,nx_{t-1}}$	$\rho_{y,nx}$
US Data	0.25	0.80	-0.22
BKK 94	0.22	0.61	-0.64
Dynare	0.28	0.59	-0.82

Source: Backus et al. (1994) Tables 1 and 3, and Dynare replication.

Intratemporal Substitution

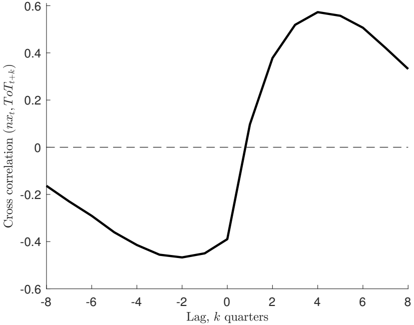
- One equilibrium equation of the model (in log-deviations):

  $$\hat{\mathcal{T}}_t = \frac{1}{\sigma}(\hat{c}_{H,t} - \hat{c}_{F,t}).$$

  provides a tight link between relative prices movements, $\mathcal{T}_t$, and the import ratio, $\hat{c}_{H,t} - \hat{c}_{F,t}$.
- We could generate a larger TOT response if we lower the substitutability of goods between H and F (smaller σ).
  - Large price movements required to make households willing to consume fewer imports.
- But even small elasticity ($\sigma = 0.5$ compared to $\sigma \approx 1.5$ in data) is not help much. See Backus et al. (1994) small elasticity case.
- Recall: Tight link between RER and TOT (Lecture 3).

Simulation Results - J-curve I

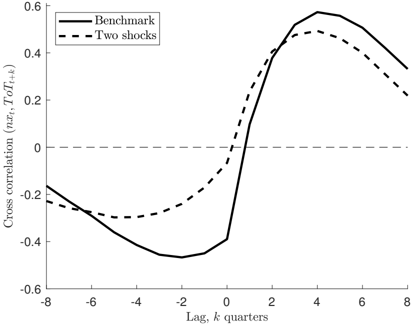
- Main point: Model can generate a J-curve.



Source: Replication of Backus et al. (1994), Figure 3.

Simulation Results - J-curve II

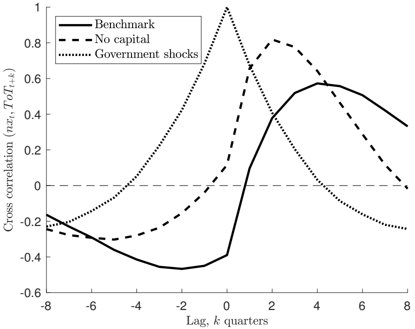
- J-curve is robust to inclusion of government spending shocks.



Source: Replication of Backus et al. (1994), Figure 7.

Simulation Results - J-curve III

- But requires productivity shocks and capital.

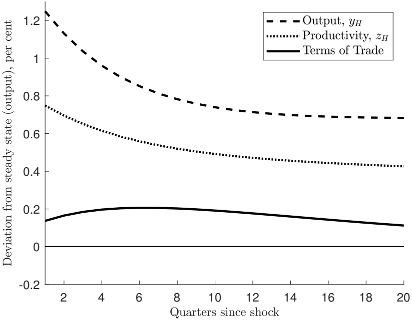


Source: Replication of Backus et al. (1994), Figure 9.

Simulation Results - Dynamics (IFRs) I

- Productivity shock at home, so immediately $z_{H,t} \uparrow$.
- In turn, this causes $y_{H,t} \uparrow$.
- Relative price of $y_{H,t}$ therefore falls, $\mathcal{T}_t \uparrow$ (depreciation).

BKK 1994, IFRs

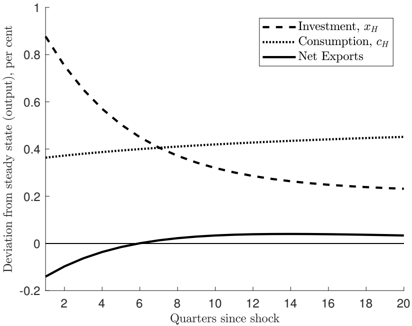


Source: Replication of Backus et al. (1994), Figure 4.

Simulation Results - Dynamics (IFRs) II

- $(c_{H,t} + x_{H,t}) \uparrow$ more than $y_{H,t} \uparrow$ such that, initially, $nx < 0$.
- Eventually, $nx > 0$ to repay the international borrowing.
- Note: $y_{H,t}$ is still more abundant, so $\mathcal{T}_t$ falls gradually.

BKK 1994, IFRs



Source: Replication of Backus et al. (1994), Figure 4.

Summary - Backus et al. (1994)

- Does well at matching basic IRBC facts.
- Builds (and to some degree improves) results from Backus et al. (1994).
- Big sucess: accounting for terms of trade movements, including reproducing a J-curve.

Subsequent Literature and Contributions

- Mendoza (1991) - SOE that can borrow and lend in international financial markets.
- Backus et al. (1992) - Two country model, single good.
- Backus et al. (1994) - Two country model with multiple goods and home bias.
    - Raffo (2008) shows that countercyclical nx in BKK only arises in nominal terms (due to ToT). Real nx are procyclical.
- Stockman and Tesar (1995) - Two country model with multiple goods and multiple sectors (add non-traded goods).
- Heathcote and Perri (2002) and Corsetti et al. (2008) - Incomplete financial markets help to solve the quantity puzzle.

RBC Models: A Four Step Solution Procedure

1. Identify all equilibrium equations and endogenous variables.
2. Use MatLab to find identify the non-stochastic steady state.
3. Use Dynare to compute dynamic solution.
4. Simulate the economy to compute business cycle facts.

Summary

- IRBC models can generate some basic features of the data.
- But they also miss several dimensions.
- In particular (at least) two large puzzles left unresolved by the classical IRBC literature.
  - Quantity puzzle. (Cross-country consumption correlation).
  - Relative price puzzle. (Volatility of relative price movements).
- Incomplete asset markets will help to resolve this (Lecture 6).
- Next lecture: nominal environment, and focus on monetary policy.

References I

Armington, P. S. (1969). A Theory of Demand for Products Distinguished by Place of Production. *IMF Staff Papers 16*(1), 159–178.

Backus, D. K., P. J. Kehoe, and F. E. Kydland (1992). International Real Business Cycles. *Journal of Political Economy 100*(4), 745–775.

Backus, D. K., P. J. Kehoe, and F. E. Kydland (1994). Dynamics of the Trade Balance and the Terms of Trade: The J-Curve? *American Economic Review 84*(1), 84–103.

Backus, D. K. and G. W. Smith (1993). Consumption and Real Exchange Rates in Dynamic Economies with Non-Traded Goods. *Journal of International Economics 35*(3-4), 297–316.

Baxter, M. (1995). International Trade and Business Cycles. In G. Grossman and K. S. Rogoff (Eds.), *Handbook of International Economics*, Volume 3, Chapter 35, pp. 1801–1864. Elsevier.

Burns, A. F. and W. C. Mitchell (1946). *Measuring Business Cycles*. New York: National Bureau of Economic Research.

Corsetti, G., L. Dedola, and S. Leduc (2008). International Risk Sharing and the Transmission of Productivity Shocks. *Review of Economic Studies* (75), 443–473.

Hamilton, J. (2017). Why You Should Never Use the Hodrick-Prescott Filter. *Review of Econics and Statistics (Forthcoming)*.

Heathcote, J. and F. Perri (2002). Financial Autarky and International Business Cycles. *Journal of Monetary Economics 49*, 601–627.

Heathcote, J. and F. Perri (2003). Why Has the U.S. Economy Become Less Correlated with the Rest of the World? *American Economic Reivew 93*(2), 63–69.

Kydland, F. E. and E. C. Prescott (1982). Time to Build and Aggregate Fluctuations. *Econometrica 50*(6), 1345–1370.

Lucas, R. E. (1975). An Equilibrium Model of the Business Cycle. *Journal of Political Economy 83*(6), 1113–1144.

Lucas, R. E. (1977). Understanding Business Cycles. In B. Karl and A. H. Meltzer (Eds.), *Carnegie-Rochester Conference Series on Public Policy*, Volume 5, pp. 7–29. Amsterdam: North-Holland.

Mendoza, E. G. (1991). Real Business Cycles in a Small Open Economy. *American Economic Review 81*(4), 797–818.

References II

Raffo, A. (2008). Net Exports, Consumption Volatility and International Business Cycle Models. *Journal of International Economics 75*, 14–29.

Schmitt-Grohé, S. and M. Uribe (2017). *Open Economy Macroeconomics*. Princeton, NJ: Princeton University Press.

Stockman, A. C. and L. L. Tesar (1995). Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovements. *American Economic Review 85*(1), 168–185.