International Economics, Lecture 3
Terms of Trade and the Real Exchange Rate

Dan Wales
University of Cambridge

London School of Economics
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Introduction

- So far: one-good economies $\rightarrow$ No exchange rate.
- Exchange rate defining feature of international macroeconomics.
  - Multiple goods $\rightarrow$ relative prices $\rightarrow$ exchange rates.
- Two key relative prices:
  - Terms of trade (TOT).
  - Real exchange rate (RER).
- Key questions:
  - What are the determinants of RER and TOT?
  - How do RER and TOT affect economic activity?

Plan

- Definitions and Derivations
- Breaking Purchasing Power Parity
- Accounting for Real Exchange Rates*
- Balassa-Samuelson Effect
- Real Exchange Rate and the Current Account

Definition I: Terms of Trade

- Consider two countries: Home (H) and Foreign (F).
  - Each country produces one differentiated good (Armington (1969) assumption).
  - Consumers in both countries like both goods.
- Let P_H be the price of the good produced in Home.
- Let P_F be the price of the good produced in Foreign.
- The **terms of trade** is defined as the relative price of **tradable goods**: the price of Foreign goods in terms of Home goods or the relative price of imports in terms of exports:

$$TOT = \frac{P_F}{P_H}.$$

  - TOT $\uparrow$ represents a **deterioration**, with an increase in the price of Foreign goods, relative to Home ones.

Definition II: Real Exchange Rate

- Let P be the price of the Home consumption bundle.
- Let P^* be the price of the Foreign consumption bundle.
- Let $\mathcal{E}$ be the nominal exchange rate.
- The **real exchange rate** is defined as the relative price of **consumption** between countries: the ratio of foreign CPI to domestic CPI expressed in a common currency:

$$RER = \frac{\mathcal{E} P^*}{P}.$$

  - RER $\uparrow$ represents a **depreciation**, with an increase in the price the Foreign consumption bundle, relative to Home.

CES Consumption Aggregator

- C_t is the total consumption basket of the representative home consumer at time t.
- The basket comprises C_H and C_F, represented by:

$$C_t = \left[\alpha_H^{1/\omega} C_{H,t}^{\frac{\omega-1}{\omega}} + \alpha_F^{1/\omega} C_{F,t}^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}},$$

  as in Anderson (1979).
- $\omega > 0$ is the elasticity of substitution between home and foreign goods = **trade elasticity**.
- $C_{H,t}$ ($C^*_{H,t}$) represents home (foreign) consumption of the home good. $C_{F,t}$ ($C^*_{F,t}$) represents home (foreign) consumption of the foreign good.
- $\alpha_H + \alpha_F = 1$.

Consumption-Based Price Index

- How much does one unit of C_t cost?
  - Consider the household's expenditure minimisation problem...
  - ... which will also yield the relative demand for each product.

Expenditure Minimisation Problem

- Define P_t (P_t^*) as the consumption-based price index for the home (foreign) economy, the price of a single unit of the aggregate consumption basket in the domestic economy, defined such that:

$$P_t \equiv \min_{C_{H,t},C_{F,t}} P_{H,t}C_{H,t} + P_{F,t}C_{F,t} \quad \text{subject to} \quad C_t = 1.$$

- Set up the problem as a Lagrangian with:

$$\mathcal{L}_t = P_{H,t}C_{H,t} + P_{F,t}C_{F,t} + \lambda_t\left(1 - \left[\alpha_H^{1/\omega} C_{H,t}^{\frac{\omega-1}{\omega}} + \alpha_F^{1/\omega} C_{F,t}^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}}\right).$$

- Two first order conditions:

$$P_{H,t} = \lambda_t \left[\alpha_H^{1/\omega} C_{H,t}^{\frac{\omega-1}{\omega}} + \alpha_F^{1/\omega} C_{F,t}^{\frac{\omega-1}{\omega}}\right]^{\frac{1}{\omega-1}} \alpha_H^{1/\omega} C_{H,t}^{-\frac{1}{\omega}},$$

$$P_{F,t} = \lambda_t \left[\alpha_H^{1/\omega} C_{H,t}^{\frac{\omega-1}{\omega}} + \alpha_F^{1/\omega} C_{F,t}^{\frac{\omega-1}{\omega}}\right]^{\frac{1}{\omega-1}} \alpha_F^{1/\omega} C_{F,t}^{-\frac{1}{\omega}}.$$

Home Consumer Demand I

- Rearrange these first order conditions find expressions for home consumer demand for the home and foreign good respectively as a function of prices, aggregate consumption and the Lagrange multiplier, λ_t:

$$C_{H,t} = \alpha_H \lambda_t^{\omega} P_{H,t}^{-\omega} C_t,$$
$$C_{F,t} = \alpha_F \lambda_t^{\omega} P_{F,t}^{-\omega} C_t.$$

- Use these demand equations in the $C_t = 1$ constraint:

$$1 = \left[\alpha_H^{1/\omega} \left(\alpha_H \lambda_t^{\omega} P_{H,t}^{-\omega} C_t \right)^{\frac{\omega-1}{\omega}} + \alpha_F^{1/\omega} \left(\alpha_F \lambda_t^{\omega} P_{F,t}^{-\omega} C_t \right)^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}},$$

$$\lambda_t^{\omega} = \left[\alpha_H P_{H,t}^{1-\omega} + \alpha_F P_{F,t}^{1-\omega} \right]^{\frac{\omega}{1-\omega}},$$

and find a functional form for the Lagrange multiplier.

Home Consumer Demand II

- Finally, calculate expenditure as:

$$
\begin{aligned}
P_t C_t &= P_{H,t} C_{H,t} + P_{F,t} C_{F,t}, \\
P_t C_t &= \alpha_H \lambda_t^{\omega} P_{H,t}^{1-\omega} C_t + \alpha_F \lambda_t^{\omega} P_{F,t}^{1-\omega} C_t, \\
P_t &= \lambda_t^{\omega} \left[\alpha_H P_{H,t}^{1-\omega} + \alpha_F P_{F,t}^{1-\omega} \right] = \lambda_t, \\
P_t &= \left[\alpha_H P_{H,t}^{1-\omega} + \alpha_F P_{F,t}^{1-\omega} \right]^{\frac{1}{1-\omega}}.
\end{aligned}
$$

- Demand for each product may then be rewritten as:

$$
\begin{aligned}
C_{H,t} &= \alpha_H \left(\frac{P_{H,t}}{P_t} \right)^{-\omega} C_t, \\
C_{F,t} &= \alpha_F \left(\frac{P_{F,t}}{P_t} \right)^{-\omega} C_t,
\end{aligned}
$$

which says that consumption of both goods is **proportional** to aggregate consumption.

Complementarity and Substitutability

- Suppose households in the Home country have CRRA utility:

$$U_t = \frac{C_t^{1-\sigma} - 1}{1-\sigma},$$

  where C_t is formed from the CES aggregator of Home and Foreign goods.

- C_H and C_F are **substitutes** if the marginal utility of one good is decreasing in the quantity of the other. Mathematically this is defined as:

$$\frac{\partial^2 U}{\partial C_H \partial C_F} = \frac{\partial^2 U}{\partial C_F \partial C_H} < 0$$

  They are **complements** if the opposite is true.

- One can show that:
  - When $\sigma\omega > 1$, the two goods are **substitutes**.
  - When $\sigma\omega < 1$, the two goods are **complements**.
  - Show this at home.

LOOP and PPP

- Law Of One Price (LOOP)
  - Price of the same good is equal in each country (after conversion to same currency units). Two goods markets here:

$$P_{H,t} = \mathcal{E}_t P^*_{H,t},$$
$$P_{F,t} = \mathcal{E}_t P^*_{F,t}.$$

- Purchasing Power Parity (PPP)
  - Price of consumption bundles is equal across countries:

$$P_t = \mathcal{E}_t P^*_t.$$

Breaking Purchasing Power Parity

- Consider the PPP relationship in this model:

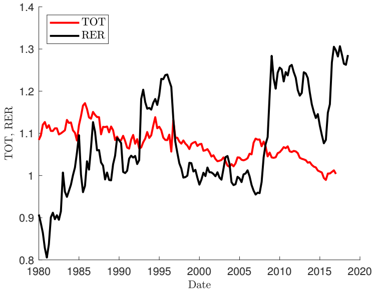
$$P_t = \left[\alpha_H P_{H,t}^{1-\omega} + \alpha_F P_{F,t}^{1-\omega}\right]^{\frac{1}{1-\omega}}$$
$$\mathcal{E}_t P_t^* = \mathcal{E}_t \left[\alpha_H^* P_{H,t}^{*,1-\omega} + \alpha_F^* P_{F,t}^{*,1-\omega}\right]^{\frac{1}{1-\omega}}.$$

- This will hold if **two conditions** are satisfied:
  1. The law of one price (LOOP) holds in both goods markets.
  2. Consumption baskets are **identical**. Specifically: $\alpha_H = \alpha_H^*$.
- Deviations from PPP therefore reflect **either** deviations from the law of one price, or preference differences across countries.
- That PPP holds is equivalent to saying $RER = 1$.

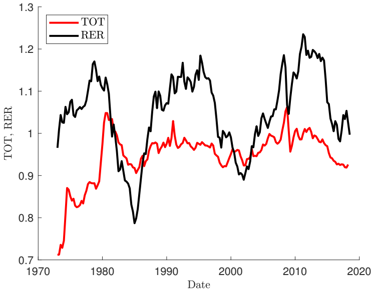
Does PPP Hold in the Data?

- RER fluctuates significantly around 1: a violation of PPP (at least in the short run).
- TOT also fluctuates around 1, but $\sigma(RER) > \sigma(TOT)$.

UK



US



Sources: Federal Reserve Board, BEA, Bank of England and ONS.

Breaking Purchasing Power Parity

- Therefore require a method to ensure PPP does not hold in the model.
- Already know two ways to break this relationship. For now, assume LOOP holds and focus instead on breaking PPP using household preferences:
  - **Home bias** in consumption.
  - **Non-tradable** goods.
- Study the relationship between RER and TOT.

Home Bias

- Consumption baskets are not assumed to be identical. Instead, consumption baskets are **symmetric**, such that:

$$\alpha_H = \alpha_F^* > 1/2.$$

- Clearly, even with LOOP, when $\alpha_H = \alpha_F^* > 1/2$:

$$P_t = \left[\alpha_H P_{H,t}^{1-\omega} + \alpha_F P_{F,t}^{1-\omega}\right]^{\frac{1}{1-\omega}}$$
$$\neq \mathcal{E}_t P_t^* = \mathcal{E}_t \left[\alpha_H^* P_{H,t}^{*,1-\omega} + \alpha_F^* P_{F,t}^{*,1-\omega}\right]^{\frac{1}{1-\omega}}.$$

- PPP **breaks down**.

A Relationship Between RER and TOT

- Definition of RER and consumption-based price indices:

$$RER^{1-\omega} \equiv \left(\frac{\mathcal{E}_t P_t^*}{P_t}\right)^{1-\omega} = \frac{\alpha_H^*(\mathcal{E}_t P_{H,t}^*)^{1-\omega} + \alpha_F^*(\mathcal{E}_t P_{F,t}^*)^{1-\omega}}{\alpha_H P_{H,t}^{1-\omega} + \alpha_F P_{F,t}^{1-\omega}}.$$

- Use home bias assumption, LOOP and then rearrange:

$$= \frac{(1-\alpha_H)P_{H,t}^{1-\omega} + \alpha_H P_{F,t}^{1-\omega}}{\alpha_H P_{H,t}^{1-\omega} + (1-\alpha_H)P_{F,t}^{1-\omega}} = \frac{(1-\alpha_H) + \alpha_H\left(\frac{P_{F,t}}{P_{H,t}}\right)^{1-\omega}}{\alpha_H + (1-\alpha_H)\left(\frac{P_{F,t}}{P_{H,t}}\right)^{1-\omega}}.$$

- Finally, note the definition of TOT:

$$= \frac{(1-\alpha_H) + \alpha_H TOT^{1-\omega}}{\alpha_H + (1-\alpha_H)TOT^{1-\omega}}.$$

Home Bias - Final Remarks

- PPP **does not hold down**, even when LOOP does.
- A relationship between RER and TOT:

$$RER^{1-\omega} = \frac{(1-\alpha_H) + \alpha_H TOT^{1-\omega}}{\alpha_H + (1-\alpha_H) TOT^{1-\omega}}.$$

- One can show that up to a first-order approximation the log-linear form of this equation with symmetry:

$$\widehat{RER}_t = (2\alpha_H - 1)\widehat{TOT}_t.$$

  around a steady state with $RER = TOT = 1$.

- The co-movement of the RER and TOT depends on the degree of home bias, α_H.
  - Positive co-movement when there is home bias, $\alpha_H > 1/2$.
  - Zero co-movement when $\alpha_H = 1/2$; this is the PPP case.
  - The co-movement is negative when $\alpha_H < 1/2$ (foreign bias).

Alternative and Complementary Assumption

- As introduced above, home bias has a limiting implication, namely:
$$corr(\widehat{RER}_t, \widehat{TOT}_t) = 1,$$
but only 0.33 in the data.

- Therefore introduce **non-tradable goods**.

Non-tradable Goods

- Introduce non-traded goods with a similar aggregator. Total consumption is now comprised of both tradable and non-tradable goods:

$$C_t = \left[\gamma^{1/\eta} C_{T,t}^{\frac{\eta-1}{\eta}} + (1-\gamma)^{1/\eta} C_{N,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}.$$

- As before, traded goods comprise home and foreign products:

$$C_{T,t} = \left[\alpha_H^{1/\omega} C_{H,t}^{\frac{\omega-1}{\omega}} + \alpha_F^{1/\omega} C_{F,t}^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}}.$$

- These are associated with the price indices:

$$P_t = \left[\gamma P_{T,t}^{1-\eta} + (1-\gamma) P_{N,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}, \quad P_{T,t} = \left[\alpha P_{H,t}^{1-\omega} + (1-\alpha) P_{F,t}^{1-\omega}\right]^{\frac{1}{1-\omega}}$$

New Form for RER

- RER with non-traded goods and symmetric preferences:

$$RER_t = \frac{\mathcal{E}_t P_t^*}{P_t} = \frac{\mathcal{E}_t P_{T,t}^*}{P_{T,t}} \cdot \frac{P_t^*/P_{T,t}^*}{P_t/P_{T,t}},$$

$$= \left[\frac{(1-\alpha)(\mathcal{E}_t P_{H,t}^*)^{1-\omega} + \alpha(\mathcal{E}_t P_{F,t}^*)^{1-\omega}}{\alpha P_{H,t}^{1-\omega} + (1-\alpha)P_{F,t}^{1-\omega}}\right]^{\frac{1}{1-\omega}}$$

$$\times \left[\frac{\gamma + (1-\gamma)(P_{N,t}^*/P_{T,t}^*)^{1-\eta}}{\gamma + (1-\gamma)(P_{N,t}/P_{T,t})^{1-\eta}}\right]^{\frac{1}{1-\eta}}.$$

- Assume LOOP in each individual traded goods market:

$$RER_t = \left[\frac{(1-\alpha) + \alpha TOT^{1-\omega}}{\alpha + (1-\alpha)TOT^{1-\omega}}\right]^{\frac{1}{1-\omega}} \times \left[\frac{\gamma + (1-\gamma)NTT_t^{*,1-\eta}}{\gamma + (1-\gamma)NTT_t^{1-\eta}}\right]^{\frac{1}{1-\eta}},$$

where $NTT_t \equiv (P_{N,t}/P_{T,t})$ and $NTT_t^* \equiv (P_{N,t}^*/P_{T,t}^*)$.

- Note: Even with LOOP in each traded goods market, when home bias exists: $P_{T,t} \neq \mathcal{E} P_{T,t}^*$.

Break Proportionality Between RER and TOT

- Up to a log-linear approximation:

$$\widehat{RER}_t = (2\alpha - 1)\widehat{TOT}_t + (1-\gamma)(\widehat{NTT}^*_t - \widehat{NTT}_t),$$

around a steady state with $TOT = NTT = NTT^* = 1$.

- This **breaks the proportionality** between $\widehat{RER}_t$ and $\widehat{TOT}_t$.
- If $\widehat{NTT}^*_t$ and $\widehat{NTT}_t$ are volatile enough, $\sigma(RER) > \sigma(TOT)$.

Accounting for Real Exchange Rates*

- The log-linear approximation provides a theoretical basis to to decompose RER movements.
- The variables of importance appear to be TOT, NTT and NTT^*. This suggests the following regression:

$$\Delta\widehat{RER}_t = \beta_1 \Delta\widehat{TOT}_t + \beta_2(\Delta\widehat{NTT}^*_t - \Delta\widehat{NTT}_t),$$

  where $\beta_1 = 2\alpha - 1$ and $\beta_2 = 1 - \gamma$ and Δ required due to unit roots.
- After computing β_1 and β_2 we can decompose variance:

$$var[\Delta\widehat{RER}_t] = \beta_1^2 var[\Delta\widehat{TOT}_t] + \beta_2^2 var[(\Delta\widehat{NTT}^*_t - \Delta\widehat{NTT}_t)] + \beta_1\beta_2 cov[\Delta\widehat{TOT}_t, (\Delta\widehat{NTT}^*_t - \Delta\widehat{NTT}_t)],$$

Engel (1999)*

- Engel (1999) takes this idea to the data.
  - Difficulty separating $P_{T,t}$ and $P_{N,t}$, so uses wide range of plausible estimates.
  - Concludes $cov[\Delta \widehat{TOT}_t, (\Delta \widehat{NTT}^*_t - \Delta \widehat{NTT}_t)] \approx 0$, and hence:

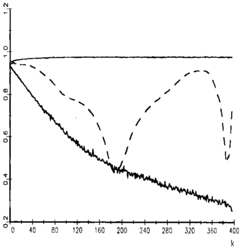
$$var[\Delta \widehat{RER}_t] \approx \beta_1^2 var[\Delta \widehat{TOT}_t] + \beta_2^2 var[(\Delta \widehat{NTT}^*_t - \Delta \widehat{NTT}_t)].$$

  - Computes fraction of $var[\Delta \widehat{RER}_t]$ explained by $var[\Delta \widehat{TOT}_t]$.

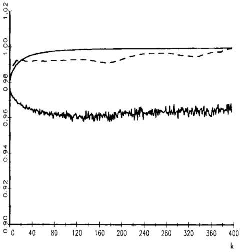
Results*

- TOT found to explain most of RER volatility.
- Relative price of non-traded goods accounts for virtually none of the RER movements.

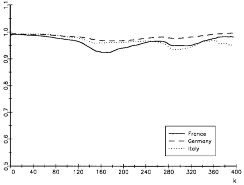
Canada



Japan



France, UK & Germany



Source: Engel (1999). For Canada and Japan, dashed lines show proportion explained by $var[\Delta \widehat{TOT}_t]$ at different time horizons, k. 95% confidence intervals also shown. For France, UK & Germany only proportion shown.

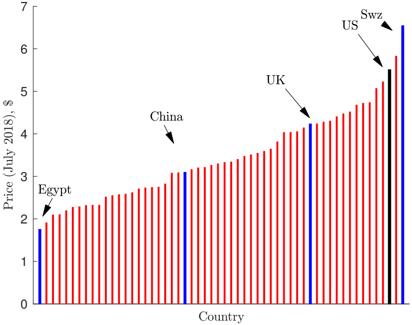
Subsequent Literature*

- Conclusion robust to alternative RER and TOT measures.
- Confirmed by Chari et al. (2002) who generate real exchange rate shocks using a sticky-price model.
- Qualifications, Betts and Kehoe (2006):
  - Data frequency does not appear to matter.
  - De-trending techniques, price series and trading partners appear to matter.

Classic Example of Breaking PPP

- International variation in the price of a McDonalds Big Mac highlights how PPP **does not hold** in the data.
- Here, the price of a Big Mac may be used to indicate whether a currency is over- or under-valued.

International Big Mac Prices.



Source: The Economist.

Harrod-Balassa-Samuelson Effect

- Model follows the seminal contributions by Balassa (1964) and Samuelson (1964). Application of a specific-factor model.
- Consider a two sector small open economy that produces:
  - Tradable goods, T.
  - Non-tradable goods, N.
- Production function for tradables (T) and non-tradables (N):

$$y_{T,t} = A_{T,t} k_{T,t}^{\alpha_T} \ell_{T,t}^{1-\alpha_T} \quad \text{and} \quad y_{N,t} = A_{N,t} k_{N,t}^{\alpha_N} \ell_{N,t}^{1-\alpha_N},$$

  where $0 < \alpha_i < 1$ and $\alpha_T \geq \alpha_N$ such that the non-traded goods sector is more labour intensive.

Factors of Production

- Perfect competition in goods and factor markets.
- Distinguish between factors of production:
  - Labour, ℓ_t, is mobile between sectors:

$$\ell_t = \ell_{T,t} + \ell_{N,t}.$$

  - This will equalise the real wage across each sector:

$$w_t = w_{T,t} = w_{N,t}.$$

  - Capital, k_t, is mobile between sectors **and countries**.

$$k_t = k_{T,t} + k_{N,t}.$$

  - This will equalise the real interest rate across each sector **and country**:

$$r_t = r_{T,t} = r_{N,t}.$$

  - Where w_t is the real wage and r_t is the real interest rate. Both are written in terms of tradables.

Firm's Problem

- Let $P_{T,t} = 1$ be the numeraire.
- Relative price of non-tradables is then $p_t \equiv \frac{P_{N,t}}{P_{T,t}} = P_{N,t}$.
- Firms maximise present value of real profits (in terms of tradables):

$$\Pi_{i,t} = \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} [P_{i,s} y_{i,s} - w_s \ell_{i,s} - i_{i,s}].$$

- Capital accumulation (from tradables) reversible and no depreciation:

$$k_{i,t+1} = k_{i,t} + i_{i,t}.$$

where $k_{i,t} > 0$ and investment, $i_{i,t}$, may be negative.

Solution to the Firm's Problem

- Firms rent capital and hire labour at rates r_t and w_t.
- The FOCs of the firm's problem are:

$$(1-\alpha_i)P_{i,t}A_{i,t}k_{i,t}^{\alpha_i}\ell_{i,t}^{-\alpha_i} - w_t = 0,$$
$$-1 + \frac{1}{1+r}[\alpha_i P_{i,t+1}A_{i,t+1}k_{i,t+1}^{\alpha_i-1}\ell_{i,t+1}^{1-\alpha_i} + 1] = 0.$$

- Taken together we have:

$$w_t = (1-\alpha_T)A_{T,t}\left(\frac{k_{T,t}}{\ell_{T,t}}\right)^{\alpha_T},$$
$$w_t = (1-\alpha_N)p_t A_{N,t}\left(\frac{k_{N,t}}{\ell_{N,t}}\right)^{\alpha_N},$$
$$r = \alpha_T A_{T,t+1}\left(\frac{k_{T,t+1}}{\ell_{T,t+1}}\right)^{\alpha_T-1},$$
$$r = \alpha_N p_{t+1}A_{N,t+1}\left(\frac{k_{N,t+1}}{\ell_{N,t+1}}\right)^{\alpha_N-1}.$$

Steady State I

- The SOE assumption (r exogenous) ensures we have 4 equations in 4 unknowns: $\{\frac{k_T}{\ell_T}, \frac{k_N}{\ell_N}, w, p\}$.
- We are able to completely characterise the steady state (in intensive form).
- The normalisation $P_T = 1$ ensures we can solve for w and $\frac{k_T}{\ell_T}$ directly from the tradables sector, as:

$$\frac{k_T}{\ell_T} = \left[\frac{\alpha_T A_T}{r}\right]^{\frac{1}{1-\alpha_T}},$$

$$w = (1-\alpha_T) A_T^{\frac{1}{1-\alpha_T}} \left[\frac{\alpha_T}{r}\right]^{\frac{\alpha_T}{1-\alpha_T}}.$$

such that the real wage is tightly linked to A_T.

Steady State II

- We can then take ratios of the FOC in the non-traded sector:

$$\frac{r}{w} = \frac{\alpha_N}{1-\alpha_N}\left[\frac{k_{N,t}}{\ell_{N,t}}\right]^{-1}.$$

- Use the result for w and rearrange to show:

$$\frac{k_{N,t}}{\ell_{N,t}} = \frac{\alpha_N}{1-\alpha_N}\frac{w}{r} = \frac{\alpha_N(1-\alpha_T)}{1-\alpha_N}\left[\frac{A_T}{r}\right]^{\frac{1}{1-\alpha_T}}\alpha_T^{\frac{\alpha_T}{1-\alpha_T}}.$$

- Hence (using either of the non-tradable goods sector FOCs):

$$p = \frac{[\alpha_T^{\frac{\alpha_T}{1-\alpha_T}}(1-\alpha_T)]^{1-\alpha_N}}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}} \cdot \frac{A_T^{\frac{1-\alpha_N}{1-\alpha_N}}}{A_N} r^{\frac{\alpha_N-\alpha_T}{1-\alpha_T}}.$$

Log-Deviations from Steady State

- Take logs of the final equation:

$$\ln p = \Gamma + \frac{1-\alpha_N}{1-\alpha_T} \ln A_T - \ln A_N + \frac{\alpha_N - \alpha_T}{1-\alpha_T} \ln r.$$

where $\Gamma \equiv \ln\{[\alpha_T^{\frac{\alpha_T}{1-\alpha_T}}(1-\alpha_T)]^{1-\alpha_N}/\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}\}$.

- Log-deviations from steady state ($\hat{x}$) may be written as:

$$\hat{p} = \frac{1-\alpha_N}{1-\alpha_T}\hat{A}_T - \hat{A}_N + \frac{\alpha_N - \alpha_T}{1-\alpha_T}\hat{r}.$$

Relative Prices and Relative Productivity

- For now, assume r is fixed (set on international markets).

$$\hat{p} = \frac{1-\alpha_N}{1-\alpha_T}\hat{A}_T - \hat{A}_N.$$

- Suppose $\hat{A}_T > \hat{A}_N > 0$, relative productivity growth in the traded goods sector.
- Provided $\alpha_T > \alpha_N$, then $\hat{p} > 0$ and the relative price of non-traded goods increases.
- What happened?
  - As productivity increases, the real wage increases.
  - Perfect competition ensure this affects both sectors.
  - The traded goods sector is more labour intensive, and therefore draws a greater (relative) level of labour.
  - Given constant real interest rate, prices in the labour-intensive sector therefore increase more quickly. $\hat{p}$ increases.

Relative Prices and the Real Interest Rate

- Now assume productivity is fixed, then:

$$\hat{p} = \frac{\alpha_N - \alpha_T}{1 - \alpha_T} \hat{r}.$$

- Suppose $\hat{r} > 0$, an increase in the real interest rate.
- Provided $\alpha_T > \alpha_N$, then $\hat{p} < 0$ and the relative price of non-traded goods falls.
- What happened?
  - As the $r \uparrow$, the real wage falls in both sectors.
  - Again, greater labour intensity in the traded goods sector results in a greater (relative) level of labour.
  - Given constant productivity levels, prices in the labour-intensive sector therefore fall more quickly. $\hat{p}$ falls.

Alternative Mathematical Route*

- Start with the FOCs. Recall:

$$w_t = (1-\alpha_T)A_{T,t}\left(\frac{k_{T,t}}{\ell_{T,t}}\right)^{\alpha_T}, \quad w_t = (1-\alpha_N)p_t A_{N,t}\left(\frac{k_{N,t}}{\ell_{N,t}}\right)^{\alpha_N},$$

$$r = \alpha_T A_{T,t+1}\left(\frac{k_{T,t+1}}{\ell_{T,t+1}}\right)^{\alpha_T-1}, \quad r = \alpha_N p_{t+1} A_{N,t+1}\left(\frac{k_{N,t+1}}{\ell_{N,t+1}}\right)^{\alpha_N-1}.$$

- Take logs and totally differentiate immediately:

$$\hat{w} = \hat{A}_T + \alpha_T(\hat{k}_T - \hat{\ell}_T), \qquad \hat{w} = \hat{p} + \hat{A}_N + \alpha_N(\hat{k}_N - \hat{\ell}_N),$$

$$0 = \hat{A}_T - (1-\alpha_T)(\hat{k}_T - \hat{\ell}_T), \quad 0 = \hat{p} + \hat{A}_N - (1-\alpha_N)(\hat{k}_N - \hat{\ell}_N).$$

- Substitute out factor variables, then equate using real wages:

$$\hat{A}_T + \frac{\alpha_T}{1-\alpha_T}(\hat{A}_T) = \hat{w} = \hat{p} + \hat{A}_N + \frac{\alpha_N}{1-\alpha_N}(\hat{p} + \hat{A}_N).$$

- Rearrange:

$$\hat{p} = \frac{1-\alpha_N}{1-\alpha_T}\hat{A}_T - \hat{A}_N.$$

Two Country Model

- Assume symmetric logarithmic household utility, such that price indices are given as:

$$P_t = P_{T,t}^{\gamma} P_{N,t}^{1-\gamma} \quad \text{and} \quad P_t^* = P_{T,t}^{*,\gamma} P_{N,t}^{*,1-\gamma}.$$

  where $0 < \gamma < 1$.

- By numeraire ($P_{T,t} = 1$) and LOOP in tradable goods ($P_{T,t} = \mathcal{E}_t P_{T,t}^*$) we can write the real exchange rate:

$$RER_t = \frac{\mathcal{E}_t P_t^*}{P_t} = \left(\frac{\mathcal{E}_t P_{N,t}^*}{P_{N,t}}\right)^{1-\gamma} = \left(\frac{p_t^*}{p_t}\right)^{1-\gamma}.$$

  where final equation assumes we normalise $\mathcal{E}_t = 1$.

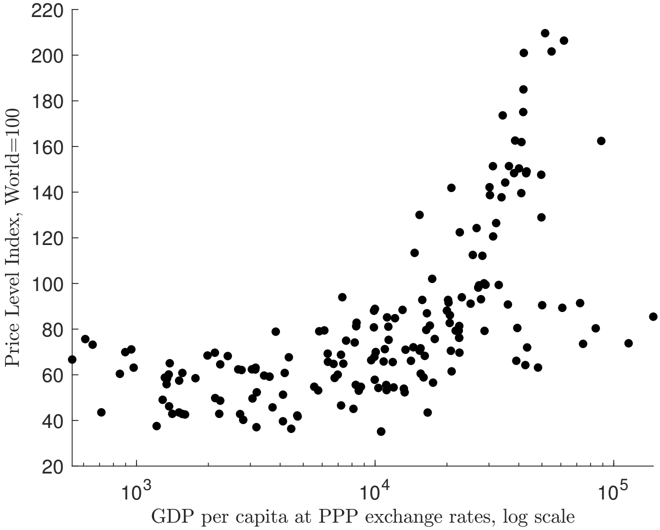
Balassa-Samuelson Effect

- SOE assumption delivers $r = r^*$, such that:

$$\widehat{RER} = (1-\gamma)(\hat{p}^* - \hat{p}) = (1-\gamma)\left[\frac{1-\alpha_N}{1-\alpha_T}(\hat{A}_T^* - \hat{A}_T) - (\hat{A}_N^* - \hat{A}_N)\right].$$

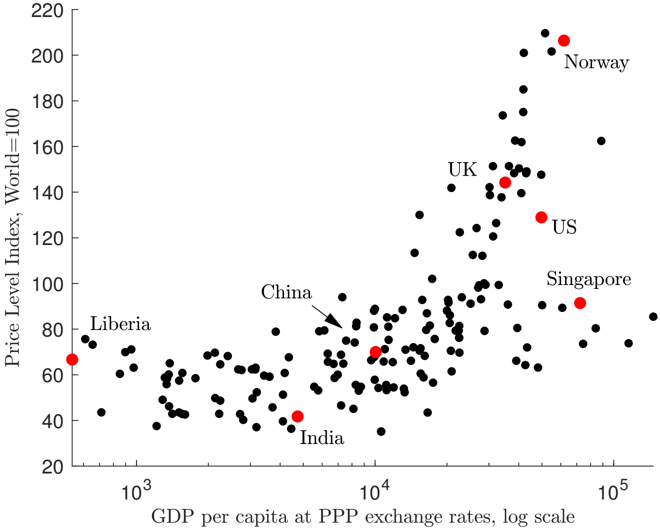
- A relative advantage in the growth rate of tradable goods $(\hat{A}_T - \hat{A}_T^*) > (\hat{A}_N - \hat{A}_N^*)$ will lead to a real exchange rate appreciation, with $\widehat{RER} < 0$. [Provided non-traded goods are relatively labour intensive $(\alpha_T > \alpha_N)$].
- Prediction: Countries with higher productivity in tradable goods, compared to non-tradable, tend to have higher aggregate price levels.
- As productivity in the non-traded goods is similar across countries (only one way to do a haircut), price levels should then be positively correlated with per-capita income.
- Countries become richer via productivity gains in the tradable sector.

Rich Countries Have High Prices I



Source: 2011 World Bank ICP, as in SGU (forthcoming).

Rich Countries Have High Prices II



Source: 2011 World Bank ICP, as in SGU (forthcoming).

Current Account Rebalancing

- What would be the consequences of a “sudden stop” US current account reversal?
- Obstfeld and Rogoff (2000, 2005) focus on implications for RER.
  - Suggest closing the US CA deficit is associated with a large real exchange rate depreciation.
- Begin with OR (2000), before discussing OR (2005).

Partial Equilibrium Analysis - Obstfeld and Rogoff (2000)

- Take RoW as given.
- Schematic:
  1. Many factors may push US CA towards historical norms. Whatever the cause, suppose suddenly closes from initial -4.4% deficit.
  2. In SR consumption of traded goods falls by -16%.
  3. The **relative** price of traded goods increases by 16%.
  4. With flexible prices and CPI stabilisation by the Fed, traded goods prices increase by 12%, non-traded goods prices fall by -4%.
  5. Exchange rate depreciates by 12%. (Pass-through dependant).

Consumption, Prices and CA

- Consumption bundle and associated price index:

$$C = \left[\gamma^{1/\theta} C_T^{\frac{\theta-1}{\theta}} + (1-\gamma)^{1/\theta} C_N^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}},$$

$$P = \left[\gamma P_T^{1-\theta} + (1-\gamma) P_N^{1-\theta}\right]^{\frac{1}{1-\theta}}.$$

- Normalise $P_T = 1$ and use demand to calculate relative price of non-traded in terms of traded goods:

$$p = \left(\frac{1-\gamma}{\gamma}\right)^{1/\theta} \left(\frac{C_T}{Y_N}\right)^{1/\theta}, \quad \rightarrow \quad P = \left[\gamma + (1-\gamma) p^{1-\theta}\right]^{\frac{1}{1-\theta}}.$$

- CA given as:

$$\frac{CA}{Y} = \frac{Y_T + rB - C_T}{Y}.$$

Calibration I - CA Reversal

2. In SR consumption of traded goods falls by -16%:

$$\frac{CA}{Y} = \frac{Y_T}{Y} + \frac{rB}{Y} - \frac{C_T}{Y}, \qquad \text{(Formula)}$$

$$-4.4\% = 25\% - 1.2\% - \frac{C_T^0}{Y}, \qquad \text{(Initial)}$$

$$0\% = 25\% - 1.2\% - \frac{C_T^1}{Y}. \qquad \text{(Final)}$$

- Hence:

$$\hat{C}_T = \ln C_T^1 - \ln C_T^0 = \ln 23.8 - \ln 28.2 = -16.9\% \approx -16\%.$$

- Data drives the choices here.

Calibration II - Prices

3. The **relative** price of traded goods increases by 16%:

$$\hat{p} = \frac{1}{\theta}\hat{C}_T = \frac{-16\%}{\theta}.$$

   Data suggests a range of $\theta \in (0.5, 4)$. For simplicity OR use $\theta = 1 \rightarrow \hat{p} = -16\%$. (Recall definition of $\hat{p}$).

4. With flexible prices and CPI stabilisation, traded goods prices increase by 12%, non-traded goods prices fall by -4%.

$$\begin{aligned} \hat{P} &= \gamma \hat{P}_T + (1-\gamma)\hat{P}_N, \\ 0\% &= 25\%[\hat{P}_N - \hat{p}] + 75\%\hat{P}_N, \\ 0\% &= 25\%[\hat{P}_N + 16\%] + 75\%\hat{P}_N, \\ \hat{P}_N &= -4\% \rightarrow \hat{P}_T = 12\%. \end{aligned}$$

Calibration III - Exchange Rate and Sensitivities

5. Exchange rate depreciates by 12%.
   - Fully-flexible prices implies P_T changes will be reflected in $\mathcal{E}$.
   - Thus $\hat{P}_T = 12\% \rightarrow \hat{\mathcal{E}} = 12\%$ (depreciation).
   - ERPT estimates of 0.5 double this: higher $\mathcal{E}$ move required for same $\hat{P}_T$.

Sensitivities

θ	1	0.5	4	1	0.5
ERPT	1	1	1	0.5	0.5
$\mathcal{E}$	12.7%	25.4%	3.2%	25.4%	50.9%

Sources: Obstfeld and Rogoff (2000) and own Calculations

Obstfeld and Rogoff (2005)

- Suggest that closing the US CA deficit (-6%) would be associated with a large real depreciation (30%).
- Updates their earlier analysis (OR (2000)).
- Use a simple, tractable, model to provide estimates:
  - Two countries, US and RoW.
  - Two sectors: tradable and non-tradable.
  - Endowment economy.

Consumption and Prices

- Consumption bundles:

$$C = \left[\gamma^{1/\theta} C_T^{\frac{\theta-1}{\theta}} + (1-\gamma)^{1/\theta} C_N^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}},$$

$$C_T = \left[\alpha^{1/\eta} C_H^{\frac{\eta-1}{\eta}} + (1-\alpha)^{1/\eta} C_F^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}.$$

- Associated price indices:

$$P = \left[\gamma P_T^{1-\theta} + (1-\gamma) P_N^{1-\theta}\right]^{\frac{1}{1-\theta}}, \quad P_T = \left[\alpha P_H^{1-\eta} + (1-\alpha) P_F^{1-\eta}\right]^{\frac{1}{1-\eta}}.$$

- LOOP for tradable goods, H and F.
- TOT ($\mathcal{T}$) and RER (Q):

$$\mathcal{T} = \frac{P_F}{P_H} = \frac{\mathcal{E} P_F^*}{\mathcal{E} P_H^*}, \quad Q = \frac{\mathcal{E} P^*}{P}.$$

Real Exchange Rate

- Assume $\alpha \neq \alpha^* \rightarrow P_T \neq \mathcal{E} P_T^*$.
- As before:
$$Q = \frac{\mathcal{E} P^*}{P} = \frac{\mathcal{E} P_T^*}{P_T} \cdot \frac{P^*/P_T^*}{P/P_T},$$
- Two components:
  1. Relative price of tradables across countries:
  $$\frac{\mathcal{E} P_T^*}{P_T} = \left[\frac{(1-\alpha^*)(\mathcal{E}_t P_{H,t}^*)^{1-\eta} + \alpha^* (\mathcal{E}_t P_{F,t}^*)^{1-\eta}}{\alpha P_{H,t}^{1-\eta} + (1-\alpha) P_{F,t}^{1-\eta}} \right]^{\frac{1}{1-\eta}} = \left[\frac{(1-\alpha^*) + \alpha^* \mathcal{T}^{1-\eta}}{\alpha + (1-\alpha)\mathcal{T}^{1-\eta}} \right]^{\frac{1}{1-\eta}}.$$
  2. Ratio of relative price of non-tradables in each country:
  $$\frac{P^*/P_T^*}{P/P_T} = \left[\frac{\gamma + (1-\gamma)(P_{N,t}^*/P_{T,t}^*)^{1-\theta}}{\gamma + (1-\gamma)(P_{N,t}/P_{T,t})^{1-\theta}} \right]^{\frac{1}{1-\theta}} = \left[\frac{\gamma + (1-\gamma) NTT_t^{*,1-\theta}}{\gamma + (1-\gamma) NTT_t^{1-\theta}} \right]^{\frac{1}{1-\theta}}.$$
  where $NTT_t \equiv (P_{N,t}/P_{T,t})$ and $NTT_t^* \equiv (P_{N,t}^*/P_{T,t}^*)$.
- In log-deviations from steady state:
$$\hat{Q} = (\alpha + \alpha^* - 1)\hat{\mathcal{T}} + (1-\gamma)(\widehat{NTT}^* - \widehat{NTT}).$$

Demand

- Recall demand functions from earlier slides. With appropriate notation, demand for traded and non-traded goods:

$$
\begin{aligned}
C_H &= \alpha\left(\frac{P_H}{P_T}\right)^{-\eta} C_T, & C_H^* &= (1-\alpha^*)\left(\frac{P_H^*}{P_T^*}\right)^{-\eta} C_T^*, \\
C_F &= (1-\alpha)\left(\frac{P_F}{P_T}\right)^{-\eta} C_T, & C_F^* &= \alpha^*\left(\frac{P_F^*}{P_T^*}\right)^{-\eta} C_T^*, \\
C_T &= \gamma\left(\frac{P_T}{P}\right)^{-\theta} C, & C_T^* &= \gamma\left(\frac{P_T^*}{P^*}\right)^{-\theta} C^*, \\
C_N &= (1-\gamma)\left(\frac{P_N}{P}\right)^{-\theta} C, & C_N^* &= (1-\gamma)\left(\frac{P_N^*}{P^*}\right)^{-\theta} C^*.
\end{aligned}
$$

Market Clearing

- Resource constraint for each good:

$$Y_H = C_H + C_H^*, \qquad Y_F^* = C_F + C_F^*,$$
$$Y_N = C_N, \qquad Y_N^* = C_N^*.$$

- Use demand functions, multiply by P_j and P_j^* and use LOOP:

$$P_H Y_H = \alpha\left(\frac{P_H}{P_T}\right)^{1-\eta} P_T C_T + (1-\alpha^*)\left(\frac{P_H}{\mathcal{E} P_T^*}\right)^{1-\eta} \mathcal{E} P_T^* C_T^*,$$
$$P_F Y_F^* = (1-\alpha)\left(\frac{P_F}{P_T}\right)^{1-\eta} P_T C_T + \alpha^*\left(\frac{P_F}{\mathcal{E} P_T^*}\right)^{-\eta} \mathcal{E} P_T^* C_T^*,$$
$$P_N Y_N = (1-\gamma)\left(\frac{P_N}{P}\right)^{1-\theta} PC,$$
$$\mathcal{E} P_N^* Y_N^* = (1-\gamma)\left(\frac{P_N^*}{P^*}\right)^{1-\theta} \mathcal{E} P^* C^*.$$

Current Accounts

- Assuming $\{Y_H, Y_F^*, Y_N, Y_N^*, C, C^*\}$ are exogenous.
- *CA* reversal will lead to *reallocation* of consumption.
- Current accounts:

$$CA = rB + \mathcal{E}P_H^*C_H^* - P_F C_F = P_H Y_H + rB - P_T C_T,$$

$$\mathcal{E}CA^* = \mathcal{E}P_F^* Y_F^* - rB - \mathcal{E}P_T^* C_T^* = -CA.$$

Rewrite Demand Equations in Terms of CA

- Substitute for $P_T C_T$ and $\mathcal{E} P_T^* C_T^*$.
- For traded goods, substitute directly:

$$
\begin{aligned}
P_H Y_H = {} & \alpha \left(\frac{P_H}{P_T} \right)^{1-\eta} (P_H Y_H + rB - CA) \\
& + (1 - \alpha^*) \left(\frac{P_H}{\mathcal{E} P_T^*} \right)^{1-\eta} (P_F Y_F^* - rB + CA), \\
P_F Y_F^* = {} & (1 - \alpha) \left(\frac{P_F}{P_T} \right)^{1-\eta} (P_H Y_H + rB - CA) \\
& + \alpha^* \left(\frac{P_F}{\mathcal{E} P_T^*} \right)^{-\eta} (P_F Y_F^* - rB + CA).
\end{aligned}
$$

- For non-traded goods, first eliminate PC and $P^* C^*$ using the demand equations, then substitute:

$$
\begin{aligned}
P_N Y_N &= \frac{1-\gamma}{\gamma} \left(\frac{P_N}{P_T} \right)^{1-\theta} (P_H Y_H + rB - CA), \\
\mathcal{E} P_N^* Y_N^* &= \frac{1-\gamma}{\gamma} \left(\frac{P_N^*}{P_T^*} \right)^{1-\theta} (P_F Y_F^* - rB + CA).
\end{aligned}
$$

Final Strategy

- To determine impact on Q, three unknown variables of interest (NTT, NTT^* and $\mathcal{T}$).
- Choose three equations. OR pick Home tradables, and non-tradable equations for both countries.
- Final steps:
  1. Use Home tradables to determine required impact on To determine impact on $\mathcal{T}$.
  2. Then, use non-tradable equations to determine impact on relative prices, NTT and NTT^*.
  3. Combine for impact on Q.

Equilibrium

- To match data, first rewrite the three equations of interest as:

$$1 = \frac{\alpha(1 + rb - ca)}{\alpha + (1-\alpha)\mathcal{T}^{1-\eta}} + \frac{(1-\alpha^*)(\mathcal{T}/\sigma_T - rb + ca)}{\alpha^*\mathcal{T}^{1-\eta} + (1-\alpha^*)},$$

$$\sigma_N = \left(\frac{1-\gamma}{\gamma}\right)\frac{NTT^{-\theta}(1 + rb - ca)}{[\alpha + (1-\alpha)\mathcal{T}^{1-\eta}]^{\frac{1}{1-\eta}}},$$

$$\sigma_N^* = \left(\frac{1-\gamma}{\gamma}\right)\frac{NTT^{*,-\theta}[1 - (rb - ca)\sigma_T/\mathcal{T}]}{[\alpha^* + (1-\alpha^*)\mathcal{T}^{-(1-\eta)}]^{\frac{1}{1-\eta}}},$$

where $ca \equiv CA/(P_H Y_H)$, $b \equiv B/(P_H Y_H)$, $\sigma_T \equiv Y_H/Y_F$, $\sigma_N \equiv Y_N/Y_H$ and $\sigma_N^* \equiv Y_N^*/Y_F^*$.

Experiment and Calibration

- Calibrate model to match CA/GDP = -6%.
- Experiment: How much will Q respond if $CA/GDP \to 0$?

Quantities and Parameters

Variable	Target/Source	Calibration
Current account	$CA/GDP \approx -6\%$	$ca = -0.2$
Net foreign assets	$B/GDP \approx 25\%$	b=-0.8
Real interest rate	$r = 5\%$	r=0.05
Rel. size of US tradables	$\frac{P_H Y_H}{P_H Y_H + P_F Y_F^*} = 22\%$	$\sigma_T = 0.22$
Rel. size of US non-tradables	$Y_N/Y_H = 1$	$\sigma_N = \sigma_N^* = 1$
Tradables share of income	25%	$\gamma = 0.25$
Tradables elasticity	Literature	$\eta = 2$
Non-tradables elasticity	Literature	$\theta = 1$
Home bias (H)	Literature	$\alpha = 0.7$
Home bias (F)	Literature	$\alpha^* = 0.925$

Source: Obstfeld and Rogoff (2005).

Results and Robustness

- First case: Over double the impact of initial paper.
- ToT change reinforces impact of relative prices on RER
- Final case: No ToT adjustment, RER moves c. 50% as much.

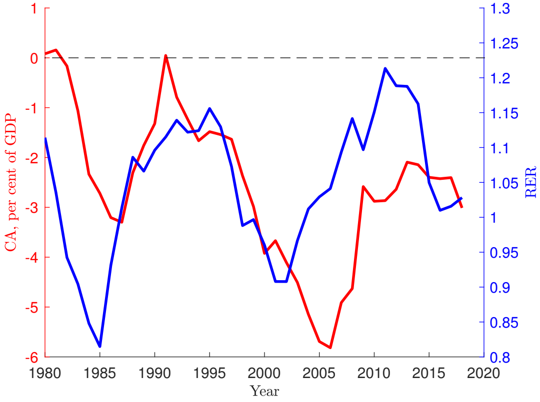
Results

θ	η	$\hat{\mathcal{T}}$	$\hat{Q}$
1	2	15.8	32.3
1	3	9.4	26.4
2	2	15.8	19.1
2	3	9.4	14.4
0.5	2	15.8	64.4
1	1000	0	17.6

Source: Obstfeld and Rogoff (2005), Table 1.

What's Happened Since Obstfeld and Rogoff (2005)?

- CA/GDP moved from -5.8% in 2005 to -2.1% in 2013.
- Simultaneously RER depreciated by 15.3%.



Sources: WEO and Federal Reserve.

Limitations

- OR's calculations broadly consistent with data.
- Yet, theoretically some unpleasant assumptions:

1. Fixing endowments means no quantity adjustment:
   - Rebalancing occurs over very short run (sudden stop).
   - No factor reallocation across sectors.
2. Sanguine view of "deep" US capital markets:
   - In practise, though CA deficit is only 6% of total US production it represents over 20% of traded good production.
3. Assume complete pass-through of dollar price movements.
   - Incomplete pass-through would suggest CA adjustment associated with a larger depreciation.

Conclusions

- Introduced multiple goods and key relative prices:
  - Terms of trade.
  - Real exchange rate.
- Real exchange rate volatile, and PPP fails empirically.
- Relative price of non-tradables is an important source of fluctuations:
  - Still under much debate.
  - Determined by productivity differentials across countries and sectors.
- Real exchange rate depreciations and US CA rebalancing.

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