EC421: International Economics

International Macroeconomics

Additional Notes: 2 Period CA Model with Production (Graph)

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Lecture 1: 2 Period Current Account Model with Production

These additional notes verify a claim made in the Lecture 1 about the ability to read K_2 directly from the graph. For ease of exposition, the 2-period current account model with production is reproduced in Figure 1, which is reproduced along with some important intersections on both the x- and y-axis. These points represent:

A y_1 production point, free trade.

B $y_1 = c_1$ production/consumption point, financial autarky.

D c_1 consumption point, free trade.

E PPF x-axis intersection.

F c_2 consumption point, free trade.

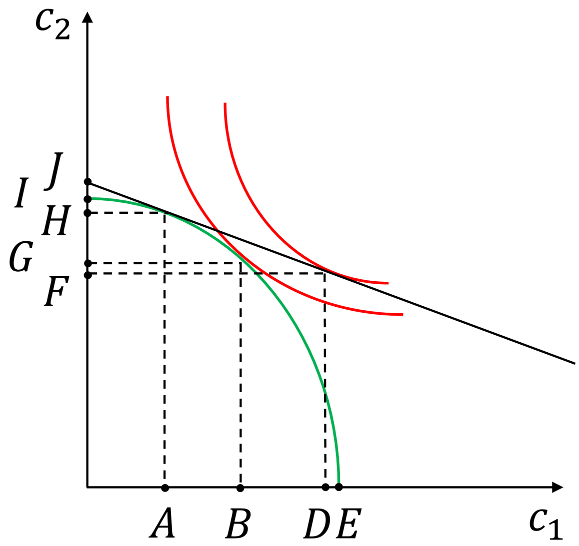
G $y_2 = c_2$ production/consumption point, financial autarky.

H y_2 production point, free trade.

I PPF y-axis intersection.

J Budget constraint y-axis intersection.

Figure 1: 2 Period CA Model with Production



For the production points, combine the period budget constraints under financial autarky to observe the Intertemporal Production Possibility Frontier (IPPF) under financial autarky:

$$c_1 = y_1 - i_1 \quad \text{and} \quad c_2 = y_2 - i_2,$$
$$c_1 = y_1 - k_2 + k_1 \quad \text{and} \quad c_2 = A_2 F(k_2) + k_2,$$
$$\text{Hence:} \quad c_2 = A_2 F(y_1 + k_1 - c_1) + y_1 + k_1 - c_1,$$

Consider two possibilities.

1. No investment in future capital: $i_1 = -k_1$. Under this scenario:

$$c_1 = y_1 + k_1,$$
$$c_2 = 0,$$
$$k_2 = 0.$$

which therefore defines the PPF intersection with the x-axis, point E.

2. Full investment in future capital: $i_1 = y_1$. Under this scenario:

$$\begin{aligned} c_1 &= 0, \\ c_2 &= A_2 F(y_1 + k_1) + y_1 + k_1 = A_2 F(k_2) + k_2, \\ k_2 &= y_1 + k_1 \end{aligned}$$

which therefore defines the PPF intersection with the y-axis, point I.

Next consider the equilibrium point (A, H). In this case we know that capital, k_2 is determined using the first order condition of the problem:

$$\begin{aligned} A_2 F'(k_2) &= r, \\ k_2 &= F'\left(\frac{r}{A_2}\right)^{-1}. \end{aligned}$$

where the inverse of the function $F'(\cdot)$ is defined as $F'(\cdot)^{-1}$ and this result displays the separation between investment and savings decisions. Hence we have that point (A, H) will be determined as:

$$\begin{aligned} c_1 &= y_1 - k_2 + k_1 = y_1 - F'\left(\frac{r}{A_2}\right)^{-1} + k_1, \\ c_2 &= A_2 F[k_2] + k_2 = A_2 F\left[F'\left(\frac{r}{A_2}\right)^{-1}\right] + F'\left(\frac{r}{A_2}\right)^{-1}, \end{aligned}$$

Therefore, the size of this (horizontal) difference between points A and E is clearly given as the distance, $k_2 = F'(\frac{r}{A_2})^{-1}$. Hence the equilibrium level of k_2, under free trade, may be read directly from the graph.