

EC421: International Economics

International Macroeconomics

Problem Set 3

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1 Consumption Bundles and Price Indexes

Consider an economy inhabited by a representative households who consumes tradable and non-tradable goods according to an overall consumption index:

$$C \equiv \left[\gamma^{\frac{1}{\theta}} C_T^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} C_N^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}},$$

with $\gamma \in (0, 1)$ and $\theta > 0$. In turn, the tradable consumption index is a CES aggregator of tradable goods produced in country H and F:

$$C_T \equiv \left[\mu^{\frac{1}{\eta}} C_H^{\frac{\eta-1}{\eta}} + (1-\mu)^{\frac{1}{\eta}} C_F^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}},$$

with $\mu \in (0, 1)$ and $\eta > 0$.

- Define the Home price of tradable goods produced in country $j = \{H, F\}$ as P_j and the price of non-tradable goods as P_N . Find the price indexes P_T and P .
- Consider a steady state in which relative prices equal one. What is the interpretation of μ and γ ?
- Show that the elasticity of substitution between Home and Foreign tradables is constant. What is it equal to?

(d) Show that:

$$\lim_{\eta \rightarrow 1} C_{Tt} = \frac{C_{Ht}^\mu C_{Ft}^{1-\mu}}{\mu^\mu (1-\mu)^{1-\mu}}$$

[Hint: Transform the aggregator so that you can apply de L'Hospital rule].

- (e) Assume that for the Foreign country $\gamma^* = \gamma$, $\theta^* = \theta$ and $\eta^* = \eta$ but $\mu^* \neq \mu$. Further assume that the law of one price holds for individual tradable goods ($P_j = P_j^*$). Does the law of one price hold for the tradable index? Suppose vice versa $\mu^* = \mu$ but $\eta^* \neq \eta$. How does your answer change?
- (f) Define the real exchange rate as $Q \equiv P^*/P$, the terms of trade as $T \equiv P_F/P_H$ and the relative price of non-tradables to tradables as $X \equiv P_N/P_T$. Is $T^* = T$? What about X^* ? Derive an expression for the real exchange rate as a function of the terms of trade and the relative price of non-tradables to tradables.
- (g) Take a log-linear approximation of the real exchange rate around a steady state in which relative prices equal one. Compute the response of the real exchange rate to a one percent increase in the terms of trade and in the relative price of non-tradables in each country.

Hint: Up to the a first order approximation, any function $f(x)$ can be written as:

$$f(x) = f(\bar{x}) + f'(\bar{x})(x - \bar{x}),$$

where \bar{x} is the point around which the approximation is taken. Using the formula for the Taylor expansion above, show that up to the first order:

$$\hat{x} \equiv \ln\left(\frac{x}{\bar{x}}\right) = \frac{x - \bar{x}}{\bar{x}}.$$

2 Dutch Disease and De-Industrialization

Consider a small open economy populated by a large number of identical households with preferences given by:

$$U(c_{T,t}, c_{N,t}) = \sum_{t=0}^{\infty} \beta^t (\ln c_{T,t} + \ln c_{N,t}), \quad (\text{A})$$

where $\beta \in (0, 1)$ and $c_{T,t}$ and $c_{N,t}$ represent the consumption of tradable and non-tradable goods, respectively. Households start period zero with initial assets d_{-1} , denominated in units of tradable goods. One unit of asset d_t pays a net interest rate r at maturity. Assume $\beta(1+r) = 1$.

The household is endowed with one unit of time. Tradable and non-tradable goods are produced with technology linear in labour:

$$y_{T,t} = A_T h_{T,t} \quad \text{and} \quad y_{N,t} = A_N h_{N,t},$$

where $y_{T,t}$ and $y_{N,t}$ denote output in the two sectors, $h_{T,t}$ and $h_{N,t}$ denote hours worked in the two sectors, and A_T and A_N denote (static) sector-specific productivity parameters. Let p_t denote the relative price of non-tradables to tradables and w_t the wage, both expressed in units of tradable goods.

- (a) Write the households' optimisation problem and derive the first order conditions.
- (b) Firms operate in perfect competition. Assuming that in equilibrium both goods are produced in positive but finite quantities, write down the solution for the real wage and the relative price of non-tradables.
- (c) Derive the solution for consumption of tradable and non-tradable goods, and for employment in the two sectors.
- (d) What is the effect of lower initial assets on consumption of both goods, and employment in both sectors?
- (e) We say that an economy "de-industrialises" (or suffers from the "Dutch disease") when the share of employment in the tradable sector falls permanently. Show that there exists a level of net foreign assets \bar{d}_{-1} above which this economy becomes completely de-industrialised.
- (f) Derive an expression for the real exchange rate (which we will define as the relative price of non-traded to traded goods, p_t) and the real wage when the economy is completely de-industrialised. Compare the solution to the previous case.