EC421: International Economics

International Macroeconomics

Problem Set 2

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1 Overlapping Generations and Production

Consider the overlapping generations model with no government discussed in Lecture 2. The economy is small with respect to the rest of the world (so the economy takes the world real interest rate r as given) and populated by overlapping generations who live for two periods (young age y and old age o). Young households supply one unit of labour inelastically in exchange for a wage w_t while old households do not work. Young households are born with no assets and can save via a competitive mutual fund, which at time t issues claims a_t that promise to pay a net interest rate r in $t+1$. The size of the cohort born in t is $N_t = (1+n)N_{t-1}$.

(a) Write down the per-period budget constraints and the intertemporal budget constraint of a representative household born in period t.

(b) A representative firm produces output y_t with a Cobb-Douglas technology

$$y_t = k_t^{\alpha}(z_t \ell_t)^{1-\alpha},$$

where $\alpha \in (0,1)$, z_t is a labour-augmenting productivity parameter that grows at rate g (i.e, $z_t = (1+g)z_{t-1}$), and k_t and ℓ_t are the amount of capital and labour, respectively, used in production. The firm rents capital from the mutual fund and hires labour from the household in competitive input markets, taking as given the factor prices r_t^k and w_t. Assume capital does not depreciate. What are the optimality conditions for capital labour? Argue that $r_t^k = r^k = r$, $\forall t$.

(c) Let b_t denote the stock of net foreign assets at time t. Write down the asset market resource constraint.

(d) Write down the labour market clearing condition.

(e) Assume utility is log in consumption and the individual discount factor is $\beta \in (0,1)$. Write down the set of equations that characterize the competitive equilibrium in efficiency units (that is, $\tilde{x}_t \equiv x_t/(z_t N_t)$ for any variable x_t in the model, except for the wage, for which $\tilde{w}_t \equiv w_t/z_t$).

(f) Solve the model (i.e., find a solution for variables in efficiency units).

(g) Derive an expression for the equilibrium current account (defined as $ca_t = b_t - b_{t-1}$) as a fraction of GDP y_t.

(h) Assume $\beta(1+r) = 1$. What can you conclude about the current account as a fraction of GDP in this model? What are the effects of a increase in the population growth rate? And of productivity growth? *[Hint: Think about reasonable values for the real interest rate to infer a range of values for β. Reasonable values for α follow from the observation that, in most countries, the labor share is between 0.7 and 0.5.]*

2 Perpetual Youth

Consider a small open economy inhabited by a continuum of identical agents of measure one who face uncertainty about their survival. In every period, independently of age, agents survive with probability γ and die (after consuming) with probability $1-\gamma$. Agents care about per-period consumption c_t according to the utility function $u(c_t)$, with $u' > 0$ and $u'' < 0$, and discount the future at rate $\beta \in (0, 1/\gamma)$.

(a) Show that expected utility for the representative agent of this economy from the perspective of time t (before consuming) is:

$$\mathbb{E}_t U_t = \sum_{j=0}^{\infty} (\beta\gamma)^j u(c_{t+j})$$

(b) Suppose that at the start of each period t a new generation, consisting again of a continuum of members of size one, is born. Show that total population size in this economy is constant and equal to $(1-\gamma)^{-1}$.

(c) Suppose a competitive insurance industry sells contracts at time t that pay agents $(1+r)/\gamma$ if they are alive at $t+1$ and nothing if they die. Agents who want to borrow can do so at the same interest rate. Show that if the insurance industry holds all of residents assets and finances all of their borrowing, earning or paying the world interest rate r, then it must break even.

(d) Agents are born with no initial wealth (no bequest motive). Argue that agents prefer to buy the insurance contracts in 3. than a standard one-period bond that pays a net interest rate r. Write down the flow budget constraint for an agent born in period ν assuming the agent receives an endowment y_t^ν and pays lump-sum taxes τ_t^ν at time t.

(e) Write the intertemporal budget constraint after imposing the relevant terminal condition.

(f) Assume that $u(c_t^\nu) = \ln c_t^\nu$. Calculate aggregate private consumption as a function of aggregate private net foreign assets, output, and taxes. Derive the law of motion of net foreign assets. *[Hint: Aggregate variables at time t are the sum from t to $-\infty$ of the variable for each cohort, weighted by the number of agents of that cohort alive at time t.]*

(g) Assume that $y_t = y$ and $\tau_t = \tau$, $\forall t$. Derive an expression that characterizes the dynamics of net foreign assets. Assuming $\beta\gamma(1+r) < 1$, describe graphically the dynamics of net foreign.

(h) Suppose government spending is equal to zero in every period but the government starts with some debt d, which it finances through a uniform tax $\tau = rd(1-\gamma)$ on everyone alive. How do changes in d affect steady state net foreign assets and consumption?